

RENSSELAER POLYTECHNIC INSTITUTE

---

# Robotic Arm Calibration and Control 6-DOF Powerball LWA 4P

---

Curtis Bradley

January 14, 2014

## 1. ACKNOWLEDGMENTS

I would like to express my great appreciation to Professor Trinkle for giving me such complete access to his lab and robot along with support for all aspects of my research. His time and generosity made this work not only possible, but enjoyable.

I am grateful to Professor Bevilacqua for helping to make this opportunity possible.

I would like to express my deepest gratitude to Bryant Pong for his tireless pursuit of capturing data and the recording, formatted, and outputting there of, without which I would not have anything to perform research on. His programming skill in various languages and ROS expertise solved problems in execution of my code I could not have hoped to solve in one semester. Thank you for all your enthusiastic coding that made this research possible.

I appreciate the technical support received from Jesse Hayes at Schunk to keep the arm running smoothly.

Thank you Jun Dong for always lending a hand with Python coding, general construction and words of encouragement.

Thank you to Shuai Li for donating his monitor as well as Python scripting help.

## 2. ABSTRACT

*The specifics to performing kinematic calibration on the Schunk Powerball Light Weight Arm (LWA) 4.6 using a serial approach starting with Circular Point Analysis (CPA) followed by a Gauss-Newton error minimization method are covered. This approach allows for a simplified regressor for the Gauss-Newton method. While this serial technique of combining these two vastly different parameter identification methods is completely developed it is not fully optimized for numerical accuracy. Additionally a solution to the inverse kinematics is completely solved and presented as it is needed for follow-on research.*

## CONTENTS

<b>1. Acknowledgments</b>	<b>2</b>
<b>2. Abstract</b>	<b>2</b>
<b>3. Introduction</b>	<b>4</b>
<b>4. Kinematics</b>	<b>5</b>
4.1. Forward Kinematics . . . . .	6
4.2. Inverse Kinematics . . . . .	8
<b>5. Control</b>	<b>11</b>
<b>6. Kinematic Calibration</b>	<b>16</b>
6.1. Circle Point Analysis Estimation . . . . .	16
6.2. Gauss-Newton Estimation . . . . .	21
<b>7. Results</b>	<b>27</b>
<b>8. Conclusion</b>	<b>28</b>
<b>A. Inverse Kinematics Matlab Code</b>	<b>30</b>
<b>B. Circular Point Analysis Matlab Code</b>	<b>35</b>
<b>C. Gauss-Newton Minimization Matlab Code</b>	<b>46</b>
<b>D. Jacobian for DH Parameters Matlab Code</b>	<b>54</b>

## LIST OF FIGURES

3.1. Powerball LWA 4.6 . . . . .	5
4.1. Nominal Powerball Zero Configuration Diagram . . . . .	7
4.2. Inverse Kinematics Example Solution . . . . .	12
5.1. System Overview . . . . .	13
5.2. Individual Joint Motion Profile . . . . .	14
5.3. Joint Angle Data . . . . .	15
6.1. Circular Point Analysis Results . . . . .	21
6.2. Gauss Newton Solution . . . . .	25
6.3. Cartesian Components of Camera Data Vs. FK . . . . .	26
6.4. 3D plot of Camera Data Vs. FK . . . . .	26
6.5. Gauss-Newton Mean Error . . . . .	27

## LIST OF TABLES

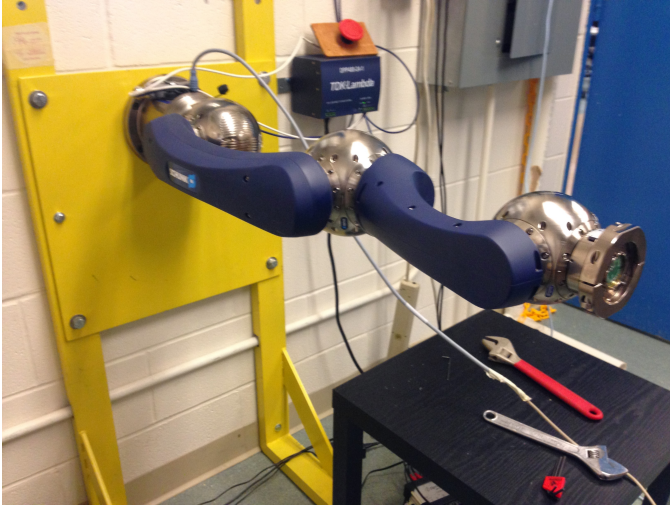
4.1. Nominal DH Parameters . . . . .	6
4.2. Test Joint Angles (radians) . . . . .	10
4.3. IK Solution Joint Angles . . . . .	11
6.1. Final DH Parameters . . . . .	24

## 3. INTRODUCTION

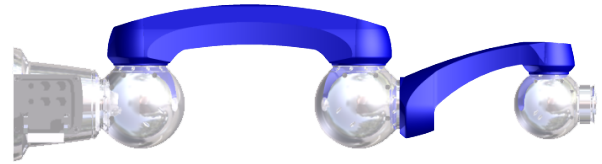
The purpose of this independent study was to gain an in-depth familiarity with a current high end robotic manipulator. The robot used was a Schunk Powerball light weight arm (LWA) 4.6, the latest 6-Degree-of-Freedom (DOF) arm from Schunk, leaders in manipulator design [1]. In order to gain this understanding two main tasks were accomplished, namely formulating the full inverse kinematics and performing the kinematic calibration. The inverse kinematics (IK) solution is necessary for fast operation everywhere in the world coordinates and also required for soft control algorithms like impedance control. The model calibrated is a parametric kinematic model using Denavit-Hartenberg(DH) parameters. The kinematic calibration is the first calibration type that makes all the proceeding calibrations more accurate, like inertial calibration and friction identification.

The Powerball is an arm designed with industry and research in mind. It has a high gear ratio for each of its axes making it insensitive to gravitational loading, but since the gearing is harmonic and the ratio is not too high it can still be back driven for impedance control and friction identification. The light compact and modular design allowed us to recover from multiple failures during initial set-up. Axis 4 failed shortly after the system was functional and cost us about a month of testing waiting for spare parts to become available. A subsequent failure of two axes, 1 and 2, would have ended research on the system for the semester, but we were able to dismantle both the original arm and a replacement arm to re-construct a completely functional arm without sending everything back to the manufacturer giving us just enough time to collect data before the end of the semester. One of the only limitations apparent so far is the low maximum speed making control schemes like minimum-time control moot. The Powerball can be used with any controller that supports multiple CANOpen devices, we chose to use the Robotic Operating System (ROS). ROS allowed us to seamlessly integrate the camera tracking system with the robot controller and will allow further extensions of the system in the future. A high resolution rendering of a model used to help simulate and visualize the Powerball, shown in figure 3.1(a).

The solution to the IK problem was solved using the nominal DH parameters determined from the manufacturers dimensions and from actuating the joints to determine offsets and axis direction. Using the nominal parameters allow work in parallel with the kinematic calibration. The solution is generally applicable inside the work envelope and includes topology informa-



(a) Schunk Powerball Lab



(b) Schunk Powerball Rendered Model

Figure 3.1: Powerball LWA 4.6

tion for the configuration of the robot since there are in general eight solutions to the Powerball IK. The IK solution also removes joint solutions from the eight if they are outside joint limits. Further there is an optional input for the last known joint position so only the closest solution to the that previous position is returned to simplify trajectory generation.

A joint trajectory using all six joints from one to six created an end effector motion profile that could be used for both Circular Point Analysis (CPA) or Gauss-Newton error minimization can be used for kinematic calibration. CPA was used to orient the end effector tracking data captured in the camera frame to the robot frame where the DH parameters are defined. Once the data is oriented Gauss-Newton error minimization can be used to directly solve for all the DH parameters at one time without adding an extra degree of freedom. Minimizing the error between the forward kinematics (FK) and the tracking data requires the solution of the symbolic version of the forward kinematics whose complication is multiplied by every additional kinematic frame. This simplification also extends to the Jacobian allowing the simplified version of these equations to be computationally faster.

#### 4. KINEMATICS

Kinematics defines the motion of bodies and in the case of a rigid body robotic manipulator it defines the relationship between joint configurations and a working reference frame. In the

case of Forward Kinematics(FK) maps joint angles to a position and orientation in the working reference frame. The Inverse Kinematics(IK) maps a given position orientation to a set of joint angles. Depending on the configuration of the robot the mapping in one direction or the another may be mathematically straight forward or there may be no closed form solution or there could be multiple solutions or no solutions at all. The FK is required for the kinematics calibration while the IK is required for advanced control algorithms and direct workspace motion planning. Robot structures are often described with an ordered list of joint types, "R" for a revolute joint and "T" for translational joint, so the Powerball is an RRRRRR robot because all six of its joints are revolute shown in figure 4.1.

#### 4.1. FORWARD KINEMATICS

Deriving the FK can be done in a variety of ways each with advantages and disadvantages, but they all start with a parameterization for the joints positions and orientations. Denavit-Hartenberg(DH) parameters are one parameterization used in a vast amount of research and industry. DH parameters have been used throughout this research to allow more straight forward comparisons to other research in the area. The DH parameters are  $\theta$ ,  $d$ ,  $a$ , and  $\alpha$ , with  $\theta$  the input variable for revolute joints and  $d$  the input variable for translational joints. The DH parameters define the transformation from one joint to the next joint in the kinematic chain,  $\theta$  defines rotation between the z-axis,  $d$  defines the distance along the z-axis between joints,  $a$  defines the distance along the x-axis between joints, and  $\alpha$  defines the angle between z-axes of joints. Each of the 6 degrees of freedom of the Powerball are a revolute joint making the  $\theta$  DH parameter the input variable for each set of four DH parameters that define every joint axis. Even though  $\theta$  is an independent input variable a DH parameter,  $\theta_{offset}$ , must still be added to  $\theta$  to form the exact FK based on the actual joint angles defined by the joint axis motor encoders. In the case of the Powerball the DH parameter representation is not unique because axes pairs one/two, three/four, and five/six are near intersecting additionally axes two and three are nearly parallel [2].

Joint Axis	$\theta_{offset}$	$d$	$a$	$\alpha$
1	0	0.205m	0	$-\frac{\pi}{2}$
2	$-\frac{\pi}{2}$	0	0.350m	$\pi$
3	$-\frac{\pi}{2}$	0	0	$-\frac{\pi}{2}$
4	0	0.305m	0	$\frac{\pi}{2}$
5	0	0	0	$-\frac{\pi}{2}$
6	0	0.075m	0	0

Table 4.1: Nominal DH Parameters

### Powerball Nominal DH Configuration

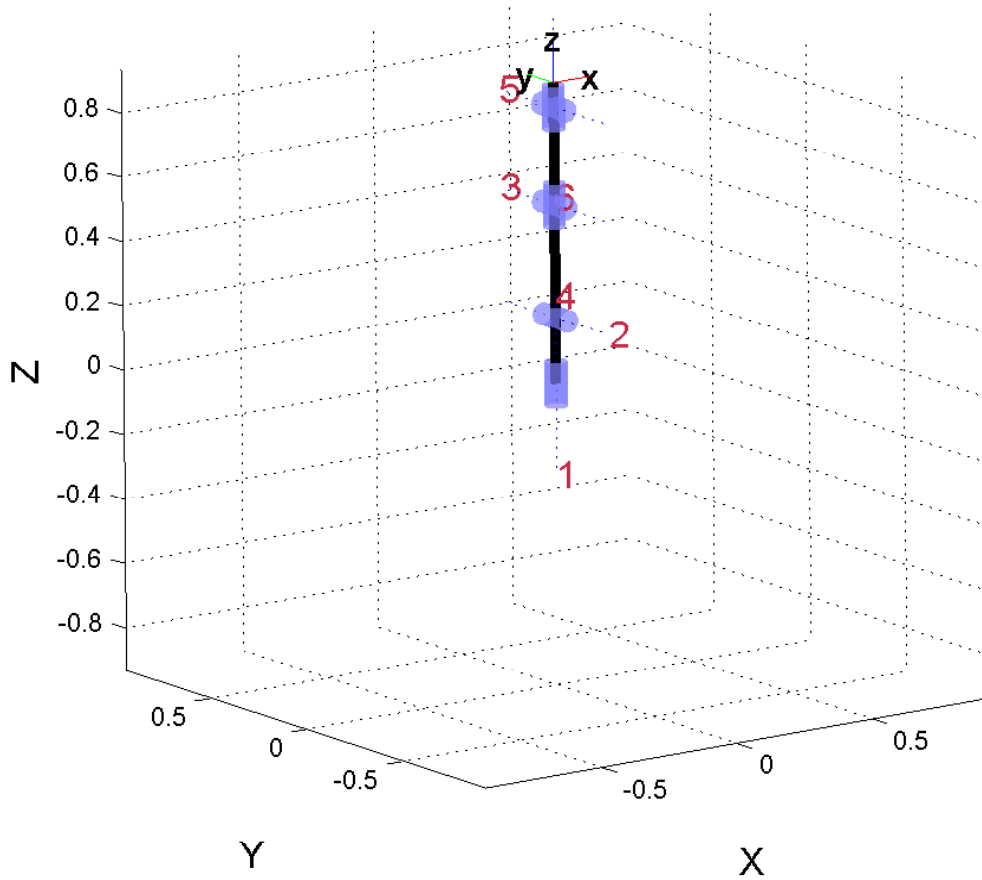


Figure 4.1: Nominal Powerball Zero Configuration Diagram

$${}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} [3] \quad (4.1)$$

Since the Powerball is composed of a serially linked set of joints the complete FK results from the straightforward matrix multiplication of each successive joint transform, for any 6-DOF serial robot that is equation 4.2.

$${}^0T_6 = {}^0T_1 {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5 {}^5T_6 \quad (4.2)$$

The 4x4 homogeneous transform matrix,  ${}^0T_6$  can be thought of as divided into an  $SO(3)$  rotation matrix,  $R_{3 \times 3}$ , a translation vector  $t_{3 \times 1}$ , a perspective vector,  $p_{1 \times 3}$ , and a scalar scale factor,  $s_{1 \times 1}$ . The composition of  ${}^0T_6$  is seen in equation 4.3. For rigid body or affine transformations that in this case represent the robot axes transformations  $p_{1 \times 3}$  equals  $0_{1 \times 3}$  and  $s_{1 \times 1}$  equals 1.

$${}^0T_6 = \begin{bmatrix} R_{3 \times 3} & t_{3 \times 1} \\ p_{1 \times 3} & s_{1 \times 1} \end{bmatrix} \quad (4.3)$$

So given all six joint values,  $[\theta_1 \dots \theta_6]$  and the correct set of twenty-four DH parameters, the FK solution is fully defined by the homogeneous transform matrix,  ${}^0T_6$  which defines the position and orientation of the end-effector in the sixth axis of the Powerball. The general formulation using all twenty-four DH parameters for  ${}^0T_6$  will resolve to a manageable complexity as there are only six variables and for most robots the nominal DH parameters have several zero entries. If the FK matrix,  ${}^0T_6$ , retains a symbolic representation for the non-zero DH parameters it will be large and if all twenty-four DH parameters retain their symbolic representation  ${}^0T_6$  becomes very large, more than 25,000 characters per line.

## 4.2. INVERSE KINEMATICS

The inverse kinematics solution (IK) will be necessary for fast direct workspace trajectory solutions and advanced control algorithms like Impedance control. The inverse kinematics problem is to solve for the joint angles give a specific robot end effector position and orientation. For a 6-DOF robot with a spherical wrist there is a closed form solution for that problem; the Powerball arm is of this class of robot. For simplicity of implementation this solution uses a homogeneous transformation matrix as an input along with an optional "previous joint" angle vector that will give the solution closest in the vector norm sense to that previous set of joint angles for easier trajectory implementation. The complete functional implementation using Matlab<sup>®</sup> is in appendix A and solves nearly instantaneously.

The outline for this solution is to solve joint 3 then joints 1 and 2 simultaneously, then solve joints 5, 4, and 6 as a set of Euler angles which have a straight forward standard solution. To solve angle 3, first  ${}^0T_6$  must be moved to the center of the spherical wrist by translating the reference frame by the distance of the constant tool vector length in the direction of the current orientation of the end-effector using equation 4.4.

$$dx = {}^0T_6(1:3, 1:3) \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix} \quad (4.4)$$



Similarly the vector from the robot base to the common center of joints 1 and 2, shoulder joint center, must also be removed from the end-effector vector yielding the vector from the center of the shoulder joint to the center of the spherical wrist from equation 4.5,

$$d_{elbow} = {}^0 T_6(1:3,4) - dx - \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}. \quad (4.5)$$

With the distance  $d_{elbow}$  all three sides of the triangle formed by the two primary sections of the Powerball arm,  $a_2$  and  $d_4$ , and the vector  $d_{elbow}$  can be used to solve the law of cosines for  $\theta_3$  as in equation 4.6,

$$\theta_3 = \pm \left( \pi - \arccos \frac{a_2^2 + d_4^2 - |d_{elbow}|^2}{2 * a_2 * d_4} \right). \quad (4.6)$$

The angle  $\theta_3$ , the "elbow", has two solutions classically termed "elbow up" and "elbow down", these solutions are independent of all other joints and so was solved for first. The topology for the solution is retained as a binary number where each binary digit stores the topology of that solution, in the case of  $\theta_3$  a 1 in the second digit means the solution has the elbow up. This allows for better trajectory planning to avoid unnecessarily jumps between topologies.

The method chosen to solve joints 1 and 2 uses the second Paden-Kahan standard subproblem and has two solutions for each of the two solutions for  $\theta_3$ , for a total of four solutions so far. The second Paden-Kahan standard subproblem also called just subproblem 2 simultaneously solves the problem of rotating a vector two consecutive rotations about two arbitrary unit vectors to another vector [4]. The arbitrary rotation unit vectors are the axes for axis 1 which is also the z axis and the other is axis 2, the y axis. To outline a solution to subproblem 2 the axes and vectors needed to solve it are shown in equation 4.7,

$$\text{axis } k_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \text{axis } k_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{p} = \begin{bmatrix} 0 \\ 0 \\ a_2 \end{bmatrix} + \begin{bmatrix} -d_4 \sin \theta_3 \\ 0 \\ d_4 \cos \theta_3 \end{bmatrix}, \quad \vec{q} = \overrightarrow{d_{elbow}}. \quad (4.7)$$

Subproblem 2 solves for the rotation of  $\vec{p}$ , the zero configuration of the vector from the shoulder to the wrist, first rotated about  $k_1$  then about  $k_2$  which will align it with  $\vec{q}$ , the actual vector from shoulder to wrist. This must be done for both  $\theta_3$  solutions. The specific numerical methods used to solve subproblem 2 are clearly labeled in appendix A as the function "subproblem 2". The two solutions are traditionally termed "shoulder left" and "shoulder right", these topologies are stored in the least significant digit, the right-most-bit, 1 for right shoulder.

With the angles for the first three joints the end-effector orientation can be rotated back to the base coordinate system which will then only represent the spherical wrist transform, also an Euler angle rotation. The  $SO(3)$  rotation matrix formed by these three rotations, roll, pitch, roll,

has a straight forward closed form solution directly from the rotation matrix,  ${}^4R_6$ , equation from the homogeneous transformation matrix,.

$${}^4R_6 = ({}^0R_1^1 R_2^2 R_3^3)^T {}^0R_6 \quad (4.8)$$

$${}^4R_6 = \begin{bmatrix} \cos\theta_4 \cos\theta_5 \cos\theta_6 - \sin\theta_4 \sin\theta_6 & -\cos\theta_6 \sin\theta_4 - \cos\theta_4 \cos\theta_5 \sin\theta_6 & -\cos\theta_4 \sin\theta_5 \\ \cos\theta_4 \sin\theta_6 + \cos\theta_5 \cos\theta_6 \sin\theta_4 & \cos\theta_4 \cos\theta_6 - \cos\theta_5 \sin\theta_4 \sin\theta_6 & -\sin\theta_4 \sin\theta_5 \\ \cos\theta_6 \sin\theta_5 & -\sin\theta_5 \sin\theta_6 & \cos\theta_5 \end{bmatrix} \quad (4.9)$$

The solution is then to use equation 4.8 on the given  ${}^0R_6$  rotation matrix portion of the transform matrix  ${}^0T_6$  with the rotation matrices for the first three joints already found. Then in the (3,3) position for the  $SO(3)$  rotation matrix from equation 4.9 is the cosine of joint angle 5. While the  $\theta_4 = \text{atan2}((2,3), (1,3))$  and  $\theta_6 = \text{atan2}((3,2), -(3,1))$ . All Euler angle transforms have 2 solutions and in this case they represent the "wrist up" and "wrist down" configurations, multiplying the total number of solutions to 8 in general, limited only by joint limits and singularities. The wrist up topology is represented by a 1 in the most significant bit, the left most bit.

To illustrate the IK solution 6 random angles are generated within the joint limits then use the FK equation 4.2 to generate the homogeneous transformation matrix,  ${}^0T_6$ , which is the input to the IK function. One set of sample joint angles and the corresponding solution are in the following tables for reference.

$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\theta_6$
-1.950	-0.717	-2.081	2.575	1.634	0.938

Table 4.2: Test Joint Angles

$${}^0T_6 = \begin{bmatrix} -0.834 & -0.118 & -0.538 & -65.739 \\ -0.200 & 0.975 & 0.095 & -56.379 \\ 0.514 & 0.187 & -0.837 & 468.762 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The complete set of 8 solutions all exist for this particular set of starting joint angles and are shown in table 4.3. The illustrated solution is shown in figure 4.2 where the actual solution is highlighted in blue and the subplot titles are the binary numbers representing the topology of each solution. Every solution shows the end-effector in the same location and orientation, but the robot uses a different set of joint angles to achieve the pose. With the Powerball this simple line drawing is difficult to see the difference because there are three pairs of intersecting axes, making left shoulder and right shoulder look identical.

	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5	Solution 6	Solution 7	Solution 8
$\theta_1$	1.191	-1.951	1.191	-1.951	1.191	-1.951	1.191	-1.9506
$\theta_2$	0.717	1.130	0.717	1.130	-1.130	-0.717	-1.130	-0.717
$\theta_3$	2.081	2.081	2.081	2.081	-2.081	-2.081	-2.081	-2.0807
$\theta_4$	-0.566	0.715	2.575	-2.427	-2.427	2.575	0.714	-0.567
$\theta_5$	1.634	2.186	-1.634	-2.186	2.186	1.634	-2.186	-1.634
$\theta_6$	0.938	-1.699	-2.203	1.443	-1.699	0.938	1.443	-2.203
Topology	111	110	011	010	101	100	001	000

Table 4.3: IK Solution Joint Angles

## 5. CONTROL

The system overview is outlined in figure 5.1. The Powerball robot arm is composed of six independently controlled servo motors each using the CANopen protocol over a common serial port using the CANopen drivers in the `ipa_canopen` ROS package. The `ipa_canopen` package includes several tools for operating a previous version of the Powerball, but are not compatible with the Powerball LWA 4.6 for reasons that are unclear but seem to be associated with package dependencies. The CANopen drivers successfully implement the portion of the protocol for sending velocity commands and receiving joint axis position and velocity feedback. Motor current or motor effort feedback is not available from the driver in ROS, but is implemented in the CANopen protocol and the joint axis controller hardware. Having state feedback and velocity control in ROS allows for several control activities, but the missing portions of the CANopen protocol preclude direct torque control, position control, and inertial calibration using joint torques. Included with the ROS package are several functions for operating and visualizing the Powerball specifically. The dependencies for these functions were not well documented, but after tracking down the build failures one at a time what seemed like all the dependencies were met without the Powerball dashboard ever working. The heart of the issue seems to be an incomplete update between versions of ROS, from Fuerte to Groovy. Ultimately only the rudimentary CANopen functions could be used in conjunction with the stand-alone diagnostic functions provided by Schunk for the Powerball.

Using the direct velocity control available with the CANopen driver in ROS in the `ipa_canopen` package changes velocity with a step function. The step changes in velocity excited structural vibrations that were higher than expected because of the combination of a light weight robot arm coupled with a flexible mounting plate. Structural vibrations add what amounts to noise to the camera tracking data since both kinematic calibration methods use a rigid body assumption for the model of the robot links. A profile with bounded jerk was used to minimize vibration. The profile uses a sinusoidal acceleration function which has a closed form solution for creating a velocity profile with a given displacement and execution time, seen in figure 5.2. The closed form solution makes the programmatic implementation of the profiles using velocity control

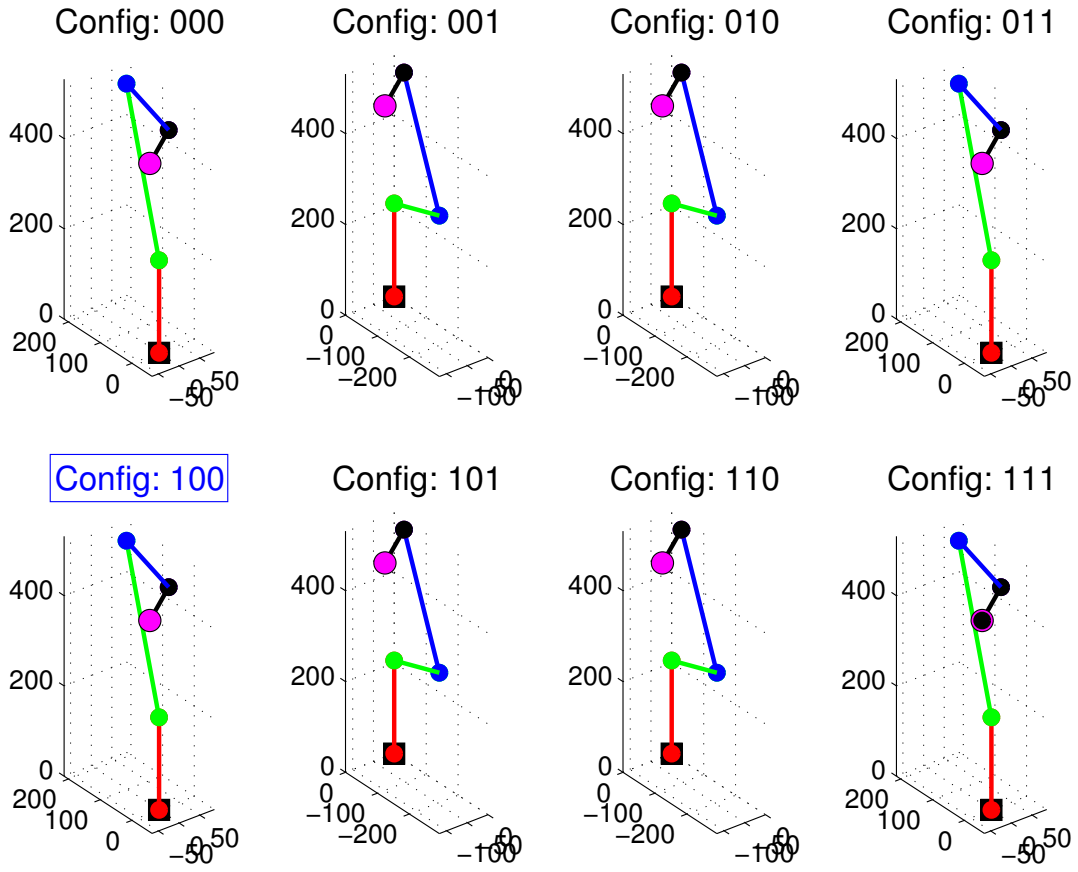


Figure 4.2: Inverse Kinematics Example Solution

possible as just a function of time in an open-loop manner. Closed loop implementations were investigated, but could not be made viable without substantial additional scripting for ROS. The open-loop method was sufficient since the only strict requirement for the motion profile was that joints be actuated one at a time. The velocity profile is generated by this equation as a function of the current ROS time initialized to zero at the start of a joint motion command,

$$velocity(t) = range Fs - range Fs \cos(t * Fs * 2 * \pi), \quad (5.1)$$

where  $Fs$  is the frequency of the motion,  $range$  is the total range of motion of the move, and  $t$  is time starting at zero to the reciprocal of  $Fs$ .

# System Overview

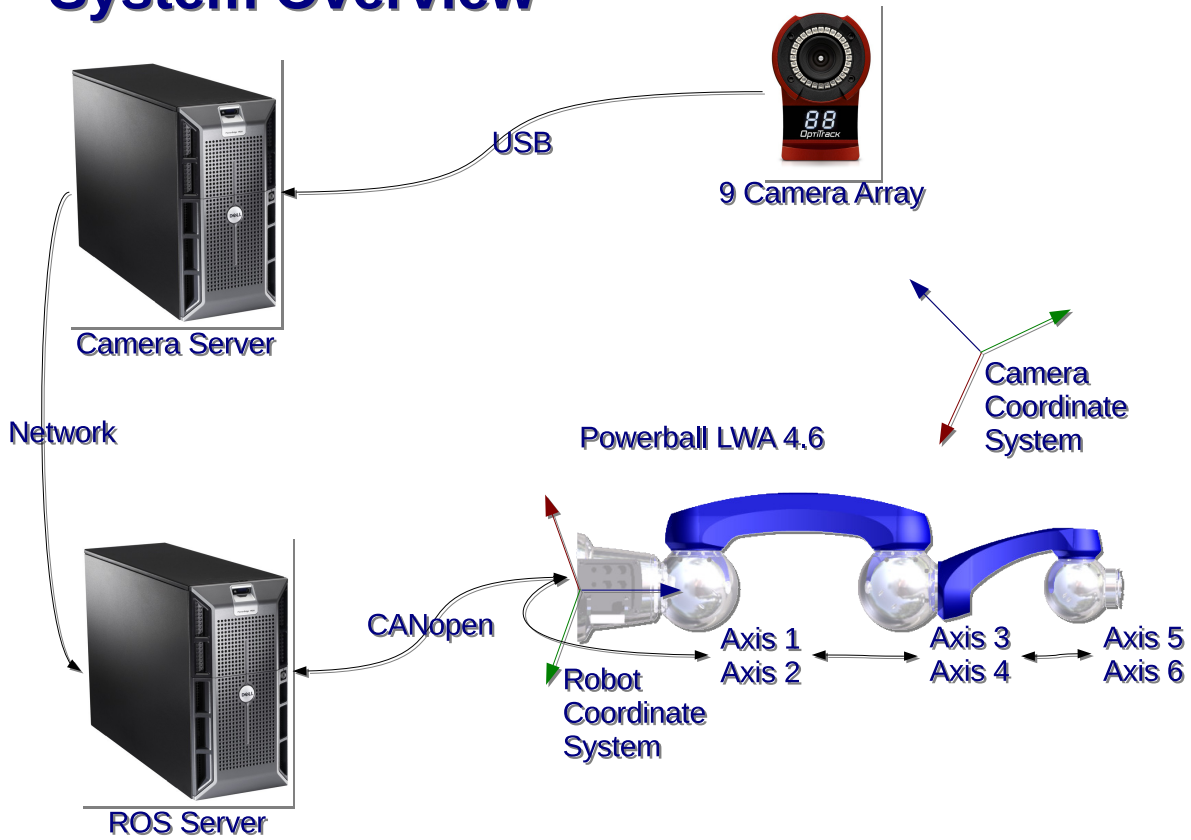


Figure 5.1: System Overview

Since the purpose of the joint actuation was to perform kinematic calibration all the joints needed to be actuated to make the DH parameters observable. The CPA kinematic calibration method requires the joint be sequentially actuated while all other joints remain fixed. In this analysis the joints were actuated from joint one to six over a range from zero to a maximum back to approximately zero. A Gauss-Newton error minimization can also be used with this joint trajectory allowing both kinematic calibration methods to be used sequentially on the same data set.

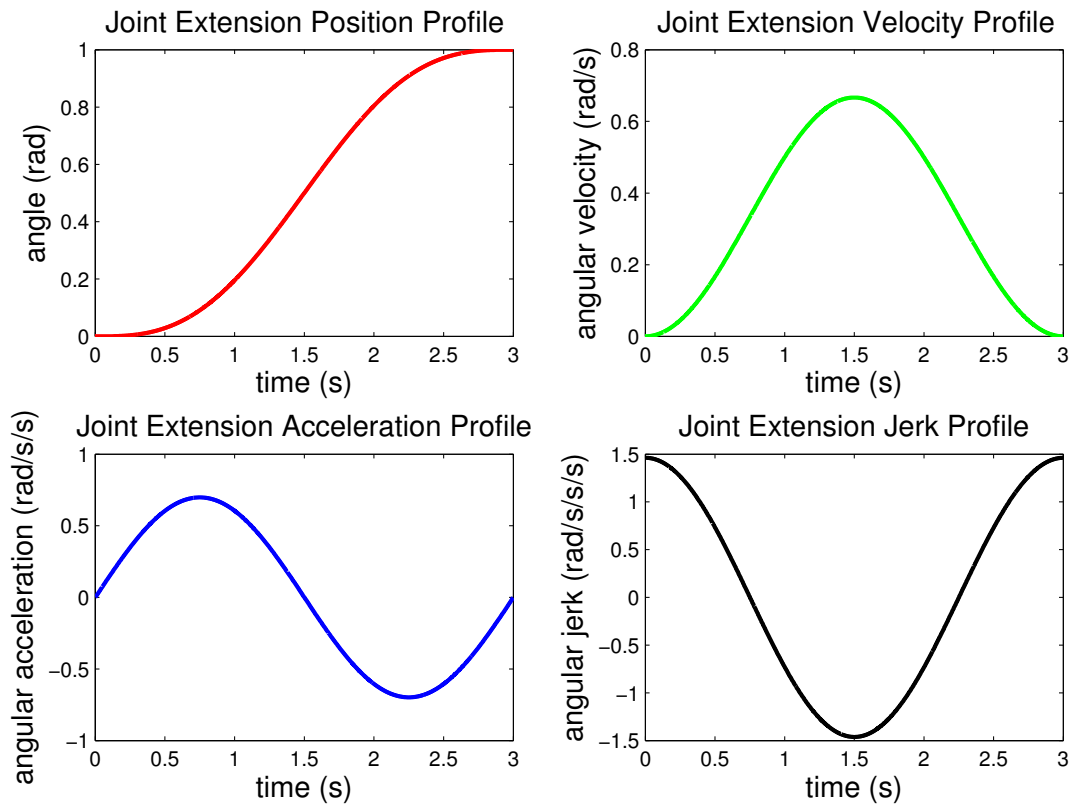


Figure 5.2: Individual Joint Motion Profile with Bounded Jerk

The system architecture of ROS uses a single central server that communicates to individual process nodes using "subscribers" and "publishers" and manages the communication between them. This server-node architecture works well with the Powerball because it acts as six separate CANopen nodes all communicating over the CAN bus additionally the Optiflex camera tracking system broadcasts its data making for easy integration as another publishing node in ROS. All the modularity comes at a price with respect to data timing and availability. During testing the subscribers that record the data for the joint angles and the subscriber that records the data for the camera tracking position are separate. To sync the two data sets absolute time the built-in ROS system time is used that is in units of nanoseconds. Despite using what should be the same time scale there is a significant delay between the joint and camera data sets that seems to be in the joint data set. The exact source of the delay could not be identified, so the exact duration was also not identified. The delay was found manually using features in the data to match the

star of motion in both the joint and camera data. Any slight error in this delay can have a large effect on the overall accuracy of a solution to the calibration problem.

Another aspect of the data timing issues is the rate difference that data can be either subscribed or published. In many situations the data is available from various processes at a rate much faster than the rate it can be accessed by the main ROS loop, making timing differences between processes much smaller than the difference between complete loop iterations times. In this set of experiments there is a high variation in when joint data is available from the Powerball modules. There was a maximum lag between readings of 1.8 seconds, and a median of 0.002s. This is compared to the camera tracking data with a maximum lag between reading 0.009s and a median of 0.0083s. This large discrepancy between data sets meant that only data that was captured at relatively the same time could be used as input data points for any calibration using joint data. Following in figure 5.3 are two plots showing entire set of over 11,000 joint data points, figure 5.3(a), and the 172 that had tight agreement with camera data in figure 5.3(b). Syncing the data would have been required even without the inconsistent timing of the joint

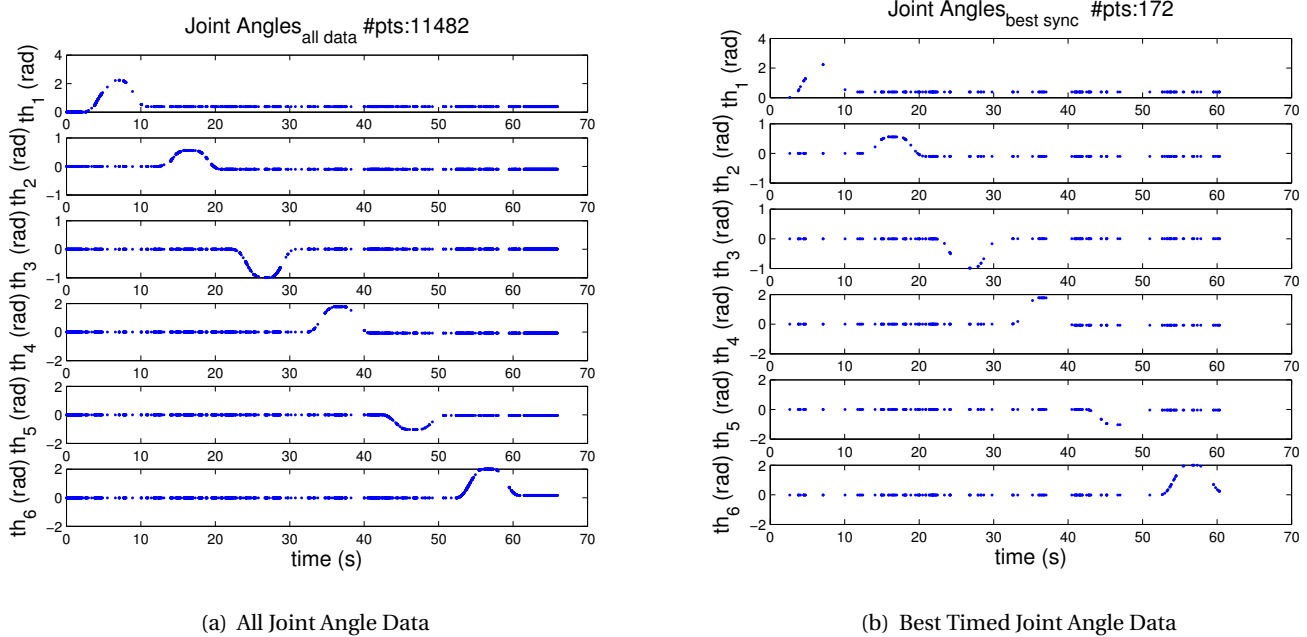


Figure 5.3: Joint Angle Data for All Six Joints Showing Timing

data because the base data rates for the camera data and the joint data are different, but what is unfortunate are the gaps in joint data seen in figure 5.3(a). Gaps in the joint data output also meant loss of control of the joint modules which caused jumps in the motion profile and unnecessary vibration of the mounting structure adding to noise in camera capture data. The

source of the gaps in joint data has not been investigated, but finding the source will be vital to future work with the Powerball arm. A loss of data for 1.8s could mean crashing the end-effector.

## 6. KINEMATIC CALIBRATION

Calibration is the process of determining the precise system parameters that affect performance in order to achieve the highest level of accuracy possible. Refining the forward kinematics is the purpose of kinematic calibration. For this project the robot involved is relatively rigid and so is well described by a parametric model using DH parameters for the FK. For robot arms the various calibration methods are divided into two main classes open loop and closed loop, where closed-loop constrains at least one degree of freedom of the end-effector motion [5], while open-loop does not [6]. Both calibration methods used, CPA and Gauss-Newton minimization, are done open-loop. Kinematic calibration was performed by using CPA to fully orient the camera data from the camera frame to the robot frame and set some of the DH parameters as the starting point of the Gauss-Newton least-squares minimization.

### 6.1. CIRCLE POINT ANALYSIS ESTIMATION

Circular point analysis (CPA) is an early method used to perform kinematic calibration because of the straight forward nature of the calculation. CPA leverages the ability to control each given joint in a known sequence while holding all other joint angles or positions fixed and tracking at least one point distal to the joint being identified [7]. In terms of DH parameters, a revolute joint has a locus of points in a circle centered at the joint z-axis in a plane perpendicular to that axis, while a translational joint has a locus of points that forms a line parallel to the z-axis. In the case of the Powerball arm all joints are revolute.

The accuracy of the CPA method for each joint is highly sensitive to the radial distance of the track point from the joint axis. This is seen in equation 6.1 where as  $r$  goes to 0 equation 6.1 becomes degenerate as the points all overlap and fail to define a plane.

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2 \quad [8] \tag{6.1}$$

One advantage of using the CPA method is that it only requires tracking discrete points and not the full pose of the end effector. This made the camera tracking process easier since tracking just the position of a body requires fewer track points than tracking the full pose of a rigid body [9]. In an attempt to extract some additional precision from the camera tracking system the orientation data was used to extrapolate the point location further from the joint axis. This showed large jumps in the position caused by orientation errors making this technique untenable for this analysis. There were two main sources of orientation error, one was the loss of track points during capture, the other was the configuration of track points was poor for orientation tracking because they formed a straight line making one axis of orientation near singular.



CPA can be performed using purely the tracking data from the camera with only the loss of the joint axis direction defined by  $\alpha$  and  $\theta_{offset}$ . Using only the tracking data for the CPA portion of the kinematic calibration allowed the elimination of the extra link reference frame typically added to align the camera reference frame to the robot reference frame, simplifying the Gauss-Newton minimization. Also the nominal values for  $\alpha$  and  $\theta_{offset}$  were sufficient as a starting point for the Gauss-Newton method.

The analysis was done one joint axis at a time, identifying first the direction then the location of the joint axis relative to the camera reference frame. First the direction of the axis was calculated using a least-squares approximation solving for the normal of the plane containing all the tracking points for that joint using the vector form of the plane equation as follows,

$$n_x(x - x_0) + n_y(y - y_0) + n_z(z - z_0) = 0 \quad [8], \quad (6.2)$$

using the centroid of the points, the arithmetic mean, as the reference point on the plain  $(x_0, y_0, z_0)$ ,

$$x_0 = \bar{x}, \quad y_0 = \bar{y}, \quad z_0 = \bar{z},$$

and then arbitrarily scaling the normal vector by assigning a value to one of the components of the normal, for instance the  $z$  direction, and incorporating all the camera data points into a matrix form then equation 6.2 becomes,

$$\begin{bmatrix} (x_1 - x_0) & (y_1 - y_0) \\ (x_2 - x_0) & (y_2 - y_0) \\ \vdots & \vdots \\ (x_n - x_0) & (y_n - y_0) \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \begin{bmatrix} (z_1 - z_0)n_z \\ (z_2 - z_0)n_z \\ \vdots \\ (z_n - z_0)n_z \end{bmatrix} \quad (6.3)$$

The normal must be scaled in the direction with the minimum range to avoid the resulting equation 6.3 from becoming singular if the normal was scaled in a direction perpendicular to the plane being solved for. Equation 6.3 is overdetermined because there are more than three equations to define  $(n_x, n_y, n_z)$ , one for each data point, this helps account for zero biased measurement noise. The solution that minimizes the  $L2$  norm between the points and the model of the plane uses the Moore-Penrose pseudoinverse of the matrix multiplying the unknowns of

the plane normal in equation 6.3. If

$$p = \begin{bmatrix} n_x \\ n_y \end{bmatrix},$$

$$A = \begin{bmatrix} (x_1 - x_0) & (y_1 - y_0) \\ (x_2 - x_0) & (y_2 - y_0) \\ \vdots & \vdots \\ (x_n - x_0) & (y_n - y_0) \end{bmatrix}, \text{ and}$$

$$q = \begin{bmatrix} (z_1 - z_0)n_z \\ (z_2 - z_0)n_z \\ \vdots \\ (z_n - z_0)n_z \end{bmatrix},$$

then the least-squares solution for  $(n_x, n_y, n_z)$  is,

$$p = A^* q, \text{ where } A^* \text{ is the pseudoinverse.} \quad (6.4)$$

With the solution of the plane parameters,  $(n_x, n_y, n_z, x_0, y_0, z_0)$ , that define the joint axis direction and the plane that contains the points defining the circle traced out by the joint the location of the axis in that plane can be solved as well. Since a circle is a subset of a sphere the locus of points for that CPA should satisfy the sphere equation, equation 6.1. Equation 6.1 is nonlinear, but a linear least-squares solution using the pseudoinverse can still be used by rearranging the equation to separate out the linear constraints by first expanding out equation 6.1 [10],

$$x^2 - 2xx_0 + x_0^2 + y^2 - 2yy_0 + y_0^2 + z^2 - 2zz_0 + z_0^2 = r^2, \quad (6.5)$$

then rearrange to collect linear, non-linear, known and unknown terms,

$$2xx_0 + 2yy_0 + 2zz_0 + (x_0^2 + y_0^2 + z_0^2 + r^2) = (x^2 + y^2 + z^2), \text{ where } (x_0^2 + y_0^2 + z_0^2 + r^2) = \zeta, \text{ then,} \quad (6.6)$$

$$\begin{bmatrix} 2x & 2y & 2z & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ \zeta \end{bmatrix} = x^2 + y^2 + z^2. \quad (6.7)$$

Now equation 6.7 can be expanded to include all the camera data points,

$$\begin{bmatrix} 2x_1 & 2y_1 & 2z_1 & 1 \\ 2x_2 & 2y_2 & 2z_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 2x_n & 2y_n & 2z_n & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ \zeta \end{bmatrix} = \begin{bmatrix} x_1^2 + y_1^2 + z_1^2 \\ x_2^2 + y_2^2 + z_2^2 \\ \vdots \\ x_n^2 + y_n^2 + z_n^2 \end{bmatrix}. \quad (6.8)$$

If we substitute into equation 6.8

$$p = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ \zeta \end{bmatrix},$$

$$A = \begin{bmatrix} 2x_1 & 2y_1 & 2z_1 & 1 \\ 2x_2 & 2y_2 & 2z_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 2x_n & 2y_n & 2z_n & 1 \end{bmatrix},$$

$$q = \begin{bmatrix} x_1^2 + y_1^2 + z_1^2 \\ x_2^2 + y_2^2 + z_2^2 \\ \vdots \\ x_n^2 + y_n^2 + z_n^2 \end{bmatrix}.$$

Then the solution of equation 6.8 uses the pseudoinverse to solve the least-squares problem the same as the plane least-squares solution in equation 6.4,

$$p = A^* q \quad (6.9)$$

This solution is degenerate because equation 6.1 defines a sphere, so the solution is ill-conditioned in the normal direction of the plane containing the circle of camera tracking points and already found in equation 6.4 as the vector  $(n_x, n_y, n_z)$ . This is not an issue because the joint axis can be defined at any point along the axis, but in order to keep the coordinates within the working envelope of the robot the point can be reflected back to the plane as follows,

$$\text{Let } N = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} [ n_x \ n_y \ n_z ],$$

then the projection equation is

$$[ x_{proj} \ y_{proj} \ z_{proj} ] = [ x_i \ y_i \ z_i ] (I - N) + [ x_0 \ y_0 \ z_0 ] N. \quad [11] \quad (6.10)$$

The solution using this CPA method shows all the axes in figure 6.1(a), but this only finds the

axis locations in the camera reference frame they were captured in which is not useful for the Gauss-Newton error minimization because the DH parameters solve the forward kinematics in the robot reference frame where the  $z_0$  axis is aligned with the  $z_{world}$  axis. The CPA solved for all the axes using over 8000 captured camera data points in 2 seconds.

With the locations of all the joint axes from the CPA method solved this information can be used to both align the  $z_0$  axis with the  $z_{world}$  axis of the data and generate a starting point for the kinematic calibration using a Gauss-Newton error minimization between the forward kinematics and the camera data aligned to the robot reference frame. The camera data can all be moved by a coordinate transformation in six DOF. Four degrees-of-freedom are needed to align the  $z$  axes, two rotation and two translation. The remaining two degrees of freedom are used to align two of the DH parameters roughly to their default values, translation to set  $d_1$  as the default height of the robot base and rotation to set  $\theta_{offset:1}$  equal to zero. Ideally these values would be determined from a geometrically suitable datum that can be directly measured and has significance for the real-world operation of the robot, like the actual mounting height of the base to define  $d_1$ . The robot operations are, as of yet undefined, so this research only deals with the accurate set-up of the robot through calibration that can be extended to suit future work.

The rotation matrix uses the normalized cross product between the identified axis 1 and the desired  $z$  axis,  $n_{cam}$ , and the  $acos$  of the dot product,  $\theta_{cam}$  to orient the data to axis 1 using the matrix exponential equation 6.11.

$$R_{cam} = e^{[n_{cam}]_x \theta_{cam}}, \quad (6.11)$$

where  $[n_{cam}]_x$  is the skew-symmetric matrix operator used to perform the cross product as a matrix multiplication. The translation offset for the data is then,

$$offset_{cam} = -R_{cam} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}. \quad (6.12)$$

To set  $d_1$  to the default value that value is used to set the location of axis 2 in aligned coordinate system. To set  $\theta_{offset:1}$  the data is simply rotated about the  $z$  axis to align the camera point where the  $\theta_1$  equals zero to the  $x$  axis to agree with the default DH parameters. The final alignment of the data showing the location of the identified axes is show in figure 6.1(b). In order to get initial estimates for two of the distance DH parameters the minimum distance between certain axes must be calculated, namely  $a_2$  between axes 2 and 3 and  $d_4$  between 3 and 5. The equation uses the normal vectors for the joints and their normalized cross product, for example,  $n_2$ ,  $n_3$  and  $n_{dist}$  with coordinates on those axes like the identified joint circle center,  $c_2$  and  $c_3$ ,

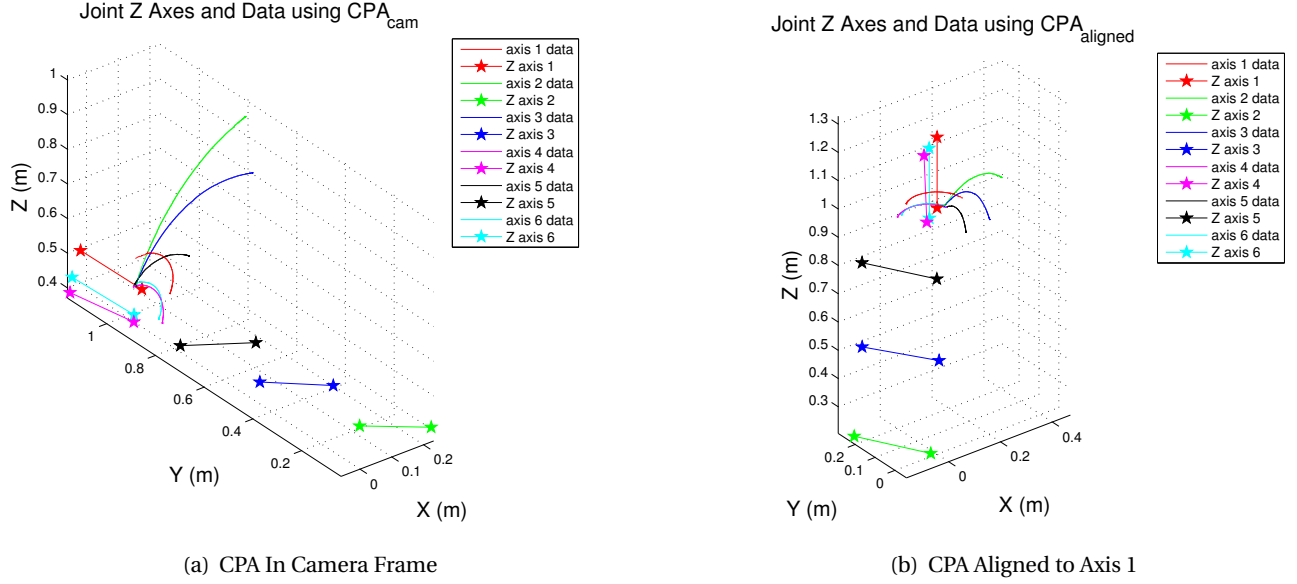


Figure 6.1: Circular Point Analysis Results

then

$$dist_1 = c_1 - \frac{((c_2 - c_1 \times n_2) \cdot n_{dist})}{(n_{dist} \cdot n_{dist})} n_1 \quad (6.13)$$

$$dist_2 = c_2 - \frac{((c_2 - c_1) \times n_1) \cdot n_{dist}}{(n_{dist} \cdot n_{dist})} n_2 \quad (6.14)$$

$$distance = norm(dist_2 - dist_1) [12]. \quad (6.15)$$

The value for  $a_2$  is derived to be  $319mm$  versus the nominal value of  $350mm$  and for  $d_4$  the derived is  $300mm$  versus  $305mm$  nominal. The source for the error between the derived and nominal cannot be known with measuring the effects of a flexible mounting, camera data error and actual robot manufacturing differences. The Matlab<sup>®</sup> code that uses CPA to align the camera data to speed and simplify the Gauss-Newton calibration is in appendix B.

## 6.2. GAUSS-NEWTON ESTIMATION

The Gauss-Newton method is an estimation method that allows the solution to a least squares minimization problem for a non-linear well defined function [13]. The method uses a linear approximation of the non-linear function formed from partial derivatives of the unknown variables, also known as a Jacobian,  $J$ . Equation 6.16 shows the structure of the non-linear para-

metric equation as a function of constant parametric values and inputs,

$$\Delta \mathbf{y}^i = f(\mathbf{x}^i, \phi), \text{ where} \quad (6.16)$$

$\mathbf{y}^i = y_1, \dots, y_n$  is the vector of all measured outputs,  $\mathbf{x}^i = x_1, \dots, x_n$  is the vector of all measured inputs, and  $\phi$  is the set of constants defining the non-linear parametric model, in this case the DH parameters [13]. For kinematic calibration the non-linear function is the FK matrix  ${}^0T_6$  or some subset of it. For this analysis just the translational portion of  ${}^0T_6$  will be used, defining the location of the end-effector with Cartesian coordinates  $f_x(\mathbf{x}^i, \phi)$ ,  $f_y(\mathbf{x}^i, \phi)$ ,  $f_z(\mathbf{x}^i, \phi)$ . This kinematics calibration solution uses the complete set of DH parameters from the forward kinematics equation which is the same for all RRRRRR robots,  $\theta_{offset}$ ,  $d$ ,  $a$ , and  $\alpha$ . Using just the position portion of the forward kinematics equations has several advantages, most notably the camera tracking system has a high degree of accuracy for a minimum set of track points where as orientation tracking requires more track points to maintain the same degree of accuracy as mentioned in the CPA subsection. All the DH parameters will be observable using just end-effector position except  $\alpha_6$ , traditionally taken as zero since it defines the arbitrary orientation of the end-effector.

Since  $f_x(\mathbf{x}^i, \phi)$ ,  $f_y(\mathbf{x}^i, \phi)$ ,  $f_z(\mathbf{x}^i, \phi)$  are nonlinear there is no closed form solution method to solve for the values of  $\phi$ , so the the Gauss-Newton or some other search method must be used. This Gauss Newton method uses a first term Taylor series expansion of equation 6.16,

$$y^i = f^i(\phi + \Delta\phi) \quad (6.17)$$

$$\approx f^i(\phi) + \left. \frac{\partial f^i(\phi)}{\partial \phi} \right|_{\phi} \Delta\phi \quad [13], \quad (6.18)$$

if we substitute into the Taylor series term in equation 6.18,  $J^i = \frac{\partial f^i}{\partial \phi}$ , also know as a regressor in this role then

$$y^i \approx f^i(\phi) + J\Delta\phi. \quad (6.19)$$

Rearranging equation 6.19 and substituting  $\Delta y = y^i - f^i(\phi)$ , the error term between the actual,  $y^i$ , and the calculated,  $f^i(\phi)$ , output yields the linearized update equation,

$$\Delta y^i = J^i \Delta\phi. \quad (6.20)$$

The superscript  $i$  indicates  $i$  equations which includes different functions  $f^i(\phi)$  and different measurements, but the least squares estimation form uses all the equations stacked in tall vec-

tors and a tall matrix and is,

$$\begin{bmatrix} x_1 - f_x^1 \\ \vdots \\ x_n - f_x^n \\ y_1 - f_y^1 \\ \vdots \\ y_n - f_y^n \\ z_1 - f_z^1 \\ \vdots \\ z_n - f_z^n \end{bmatrix} = \begin{bmatrix} x J_{\theta_{offset:1}}^1 & \cdots & x J_{\theta_{offset:6}}^1 & x J_{d_1}^1 & \cdots & x J_{d_6}^1 & x J_{a_1}^1 & \cdots & x J_{a_6}^1 & x J_{\alpha_1}^1 & \cdots & x J_{\alpha_6}^1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x J_{\theta_{offset:1}}^n & \cdots & x J_{\theta_{offset:6}}^n & x J_{d_1}^n & \cdots & x J_{d_6}^n & x J_{a_1}^n & \cdots & x J_{a_6}^n & x J_{\alpha_1}^n & \cdots & x J_{\alpha_6}^n \\ y J_{\theta_{offset:1}}^1 & \cdots & y J_{\theta_{offset:6}}^1 & y J_{d_1}^1 & \cdots & y J_{d_6}^1 & y J_{a_1}^1 & \cdots & y J_{a_6}^1 & y J_{\alpha_1}^1 & \cdots & y J_{\alpha_6}^1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ y J_{\theta_{offset:1}}^n & \cdots & y J_{\theta_{offset:6}}^n & y J_{d_1}^n & \cdots & y J_{d_6}^n & y J_{a_1}^n & \cdots & y J_{a_6}^n & y J_{\alpha_1}^n & \cdots & y J_{\alpha_6}^n \\ z J_{\theta_{offset:1}}^1 & \cdots & z J_{\theta_{offset:6}}^1 & z J_{d_1}^1 & \cdots & z J_{d_6}^1 & z J_{a_1}^1 & \cdots & z J_{a_6}^1 & z J_{\alpha_1}^1 & \cdots & z J_{\alpha_6}^1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ z J_{\theta_{offset:1}}^n & \cdots & z J_{\theta_{offset:6}}^n & z J_{d_1}^n & \cdots & z J_{d_6}^n & z J_{a_1}^n & \cdots & z J_{a_6}^n & z J_{\alpha_1}^n & \cdots & z J_{\alpha_6}^n \end{bmatrix} \begin{bmatrix} \theta_{offset:1} \\ \vdots \\ \theta_{offset:6} \\ d_1 \\ \vdots \\ d_6 \\ a_1 \\ \vdots \\ a_6 \\ \alpha_1 \\ \vdots \\ \alpha_6 \end{bmatrix} \quad (6.21)$$

Then the compact estimation form of equation 6.21 is then

$$\Delta y = J \Delta \phi. \quad (6.22)$$

The least square solution for the difference in  $\phi$  for one set of data is then

$$\Delta \phi = J^* \Delta y, \quad (6.23)$$

where  $J^*$  is the Moore-Penrose pseudoinverse. Since  $J$  is non-linear several iterations of estimates will need to be generated before  $\phi$  will converge to a solution by updating the previous estimate with  $\Delta \phi$  from equation 6.23,  $\phi^{k+1} = \phi^k + \Delta \phi$ .

For a given set of data consisting of a series of joint angles,  $\theta_{1\dots 6}^{1\dots n}$ , and corresponding camera tracking points,  $\begin{bmatrix} x^{1\dots n} \\ y^{1\dots n} \\ z^{1\dots n} \end{bmatrix}$ . Where  $n$  must be large enough so there are enough measurements

to have full rank of  $J$  meaning all parametric model parameters,  $\phi$  are observable. As mentioned earlier in this section  $\alpha_6$  is unobservable because only end-effector position is tracked, but using the 172 data points with well correlated camera and joint data mentioned in section 5  $J$  maintained full rank with  $n = 172$  as expected since the motion profile has motion for every joint. This makes the Jacobian  $J$  of size 516 by 24 for this data set.

As mentioned in section 4.1 the FK solution where the functions for the Jacobian come from are large, around 25,000 characters, so taking the partial derivatives of the functions for position,  $f_x(\mathbf{x}, \phi)$ ,  $f_y(\mathbf{x}, \phi)$ ,  $f_z(\mathbf{x}, \phi)$ , will also be large. In this case the portion of  $J$  for on data point is

over 296,000 characters. This equation would have been even larger if an additional reference frame was added to account for the camera frame as mentioned in subsection 6.1. Using the CPA method to align the data for use in the Gauss-Newton Estimation allowed the elimination of one frame that traditionally accounts for aligning the camera frame to the robot frame. Each data point requires a symbolic substitution into that large 3 by 24 subset of the Jacobian. Using the best built-in Matlab<sup>®</sup> function for this type of substitution, "subs", each data point took about 1.18s, for 174 points and 15 iterations the solution takes about 3000s making investigations into different starting points for  $\phi$  and for larger data sets prohibitively time consuming. Parallel processing multiple data points at a time only gave a marginal reduction in processing time running 12 threads on a workstation. A reduction in processing time from 1.18s to  $8\mu s$  was accomplished by computing the fully symbolic Jacobian for one data point as a function and then compiling the function to an executable "C-mex" file for direct use from Matlab<sup>®</sup>. Now a complete solution took about 45s. The complete Matlab<sup>®</sup> code that the C-mex code is generated from is in appendix D.

The combination of non-linearities, maintaining ill-conditioned DH parameters like those for parallel axes and biased errors made the direct application of the Gauss-Newton method unstable. During iterations of the solution limit-switching because of the bounded trigonometric functions, constrained to  $\pm\pi$ , would cause the error to oscillate and sometimes go unstable. To keep the angular parameters reasonable without effecting the solution the values were forced to roll-over between  $\pm 2\pi$  which produces the exact same output. The relative size of the parameters in  $\phi$  are all on the order of 1, so there are no gross scaling issues to contribute to instability issues. To stabilize the search a relaxation term was added to the regressor calculation. The relaxation term is the "golden ratio",  $\frac{2}{1+\sqrt{5}}$ . This allowed the algorithm to converge to a solution despite the numerical issues mentioned. The technique converges to different solutions depending on the initial guess. The solution to all 24 DH parameters is shown in table 6.1 while the convergence of the parameters are shown in figure 6.2.

Joint Axis	$\theta_{offset}$	$d$	$a$	$\alpha$
1	0.251	0.204m	-0.002m	4.535
2	3.600	-1.585m	0.350m	2.932
3	3.662	-1.593m	0.029m	4.761
4	0.125	0.288m	-0.027m	1.411
5	6.190	0.007m	0.013m	4.85
6	5.995	0.226m	0.090m	0

Table 6.1: Final DH Parameters

There is a large difference between the DH parameters derived from the Gauss-Newton method in table 6.1 and the initial DH parameters in table 4.1, but the improvement of fit in the derived parameters is obvious from the side-by-side comparison in figure 6.4 noting the better agreement of the camera data to the FK using the Gauss-Newton derived DH parameters than



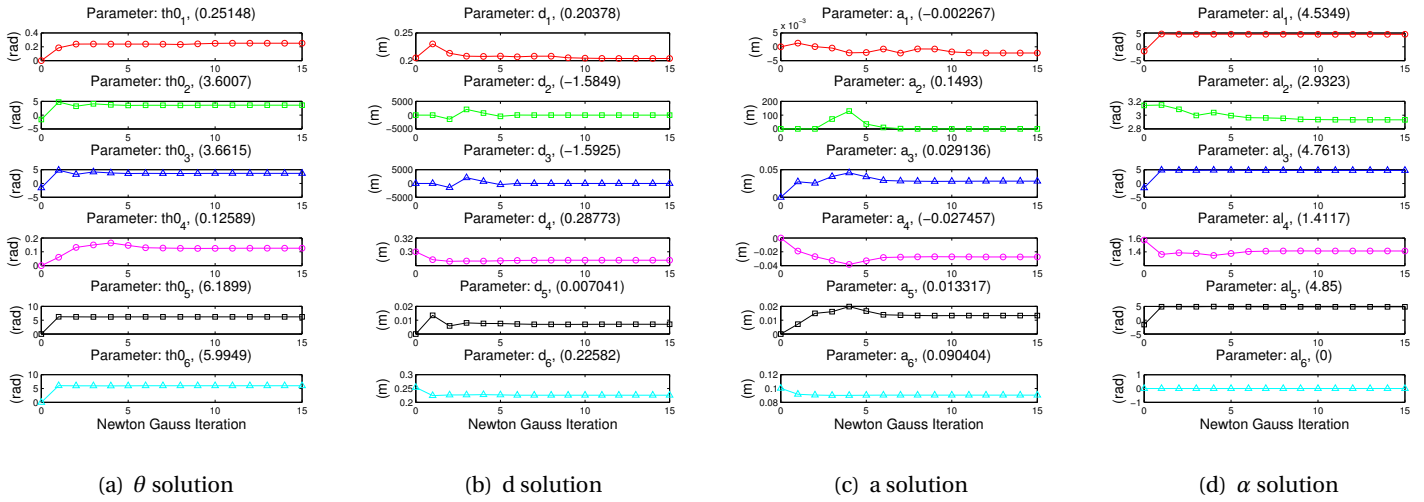
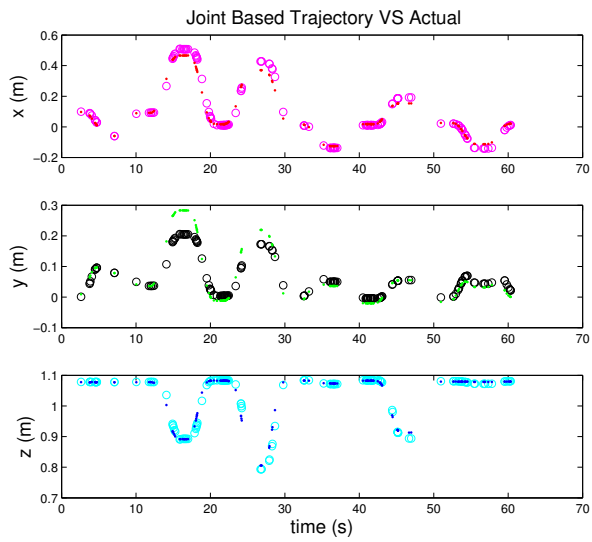


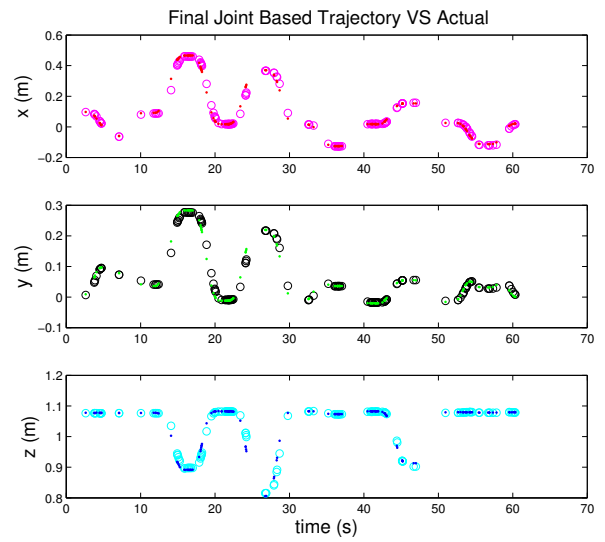
Figure 6.2: Gauss Newton Analysis Results

the initial estimate. As noted in subsection 4.1 multiple sets of DH parameters are possible for serial link mechanisms that have intersecting axes like the Powerball [2]. In figure 6.5 there is a slight "blow-up" of the solution when the error nears 200 meters around iteration 4 before converging, so it is reasonable to expect that the solution may not come back to one near the starting estimate. The final solution is very sensitive to the initial estimate of DH parameters and further study could be done to find a method that uses that sensitivity of the Gauss-Newton method or a higher order approximation to converge to the set of parameters nearest to the nominal set.

The Cartesian components of the FK using the initial DH parameters and final DH parameters are in figure 6.3, where the actual camera data are the red, green and blue "dots" and the FK calculated data are the magenta, black and cyan circles. In figure 6.3 some data points have better agreement than others ideally the plot would be a dot centered inside each circle. The timing issue discussed in section 5 may have caused joint angle data to become mistimed with camera data forcing preventing an accurate solution. The plots in figure 6.4 gives the best intuitive assessment of the fitness of the solution judging how closely the red and blue three dimensional lines overlap. Figure 6.5 shows the minimization of the error over the fifteen iterations until the stopping criterion is met, maximum absolute parameter change less than  $1e-4$ . The Matlab<sup>®</sup> code used to solve the Gauss-Newton calibration is in appendix C. The code in appendix C also calls the code in appendix D.

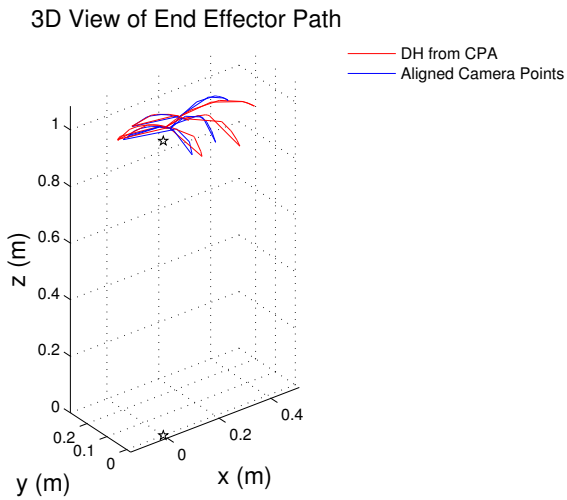


(a) Cartesian Forward Kinematics with Initial DH

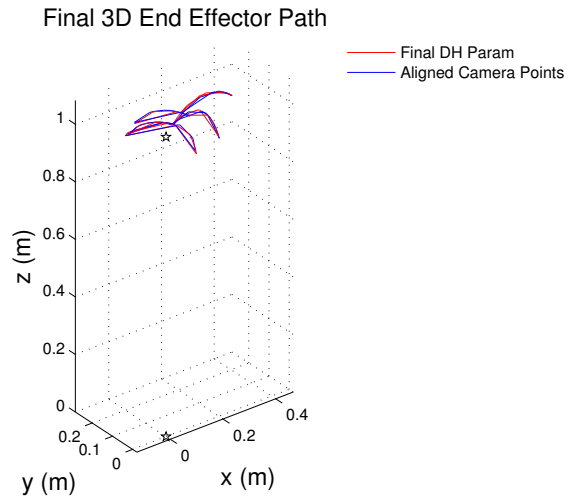


(b) Cartesian Forward Kinematics with Final DH

Figure 6.3: Cartesian Components of Camera Data Vs. FK



(a) Forward Kinematics with Initial DH



(b) Forward Kinematics with Final DH

Figure 6.4: Camera Track Points VS FK from Joint Data and DH Parameters

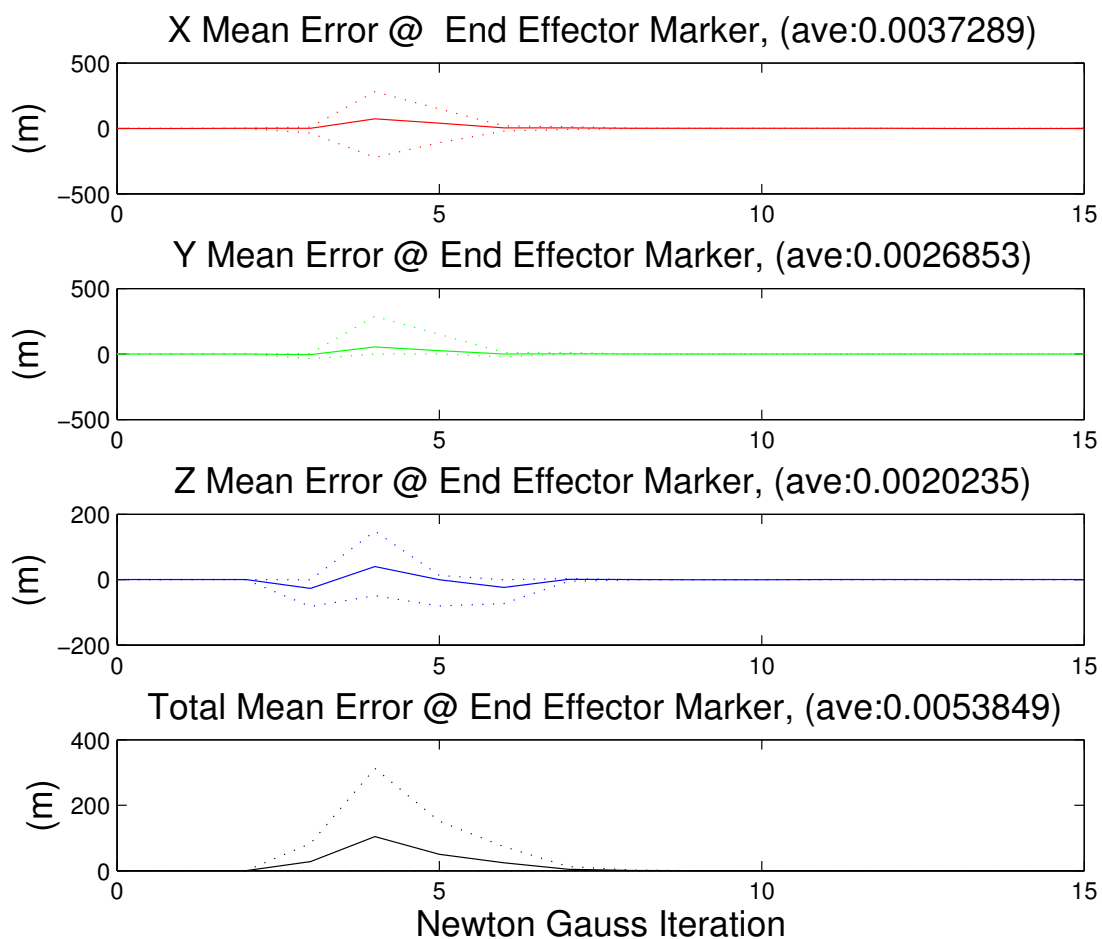


Figure 6.5: Gauss-Newton Mean Error

## 7. RESULTS

A robust solution for the IK solution that is closed form and solves fast while maintain the topology of the solution and optionally finding the solution nearest to another joint configuration in the vector norm sense. The project successfully solved for a set of DH parameters that minimized the error of the FK solution without adding an additional axis to solve for the camera reference frame but by using the CPA method to pre-align the data prior to starting the Gauss-Newton error minimization method. This solution was optimized for solution speed and modified from the standard Gauss-Newton method to relax the convergence and increase the stability making it more tolerant to initial estimates that are further from a solution. The solution

to the CPA method created for this project uses only least square solutions without having to resort to search methods or other more time intensive numerical methods.

There are issues with the absolute accuracy of the final DH parameter result that will require further testing to completely diagnose at a deeper level that stem from several sources of error. The camera configuration was non-ideal for the robot working envelope and has non-zero biased noise that is not easily removed with either the CPA or Gauss-Newton methods as they both solve for the least-square minimum errors. Similarly the support structure for the robot was flexible enough to introduce several millimeters of additional non-zero biased error to readings, this was exacerbated by the vibrations introduced by the vibrations caused by the timing delays in the motion control loop. The absolute time stamp is another issue that unnecessarily adds error the solution, but should be fairly straightforward to find the general source of the time shift though a solution may not be as forthcoming.

The current motion returned near to the zero configuration after each joint move so there were many more data points near the zero configuration biasing the result toward that location. The bias can be generally seen in figure 6.3 where the best agreement of the data are along dwell points in the motion where there are the most data points. A video of the system performing several experiments is available [here](#).

## 8. CONCLUSION

Areas of interest for research with this robot will probably need to address the timing lags from the Powerball modules. This research was able to overcome those limitations, but any type of high-speed or high accuracy actions will have serious issues. The issues with the calibration mentioned previously should be addressed and alternate motion profiles with simultaneous joint motion may have better numerical conditioning for the reasons mentioned in the 7 section. An additional point of interest will be to perform further inertial and friction calibration on the arm which will require adding CANopen Process Data Objects(PDOs) to transmit motor current or torque as the current ROS CANOpen implementation lacks access to this and other features available from the Powerball modules. For the current set of experiments packing paper was a viable, albeit unaesthetic, IR shielding method, but for future experiments requiring greater range of motion the paper will not maintain its integrity. Paint or some other semi-permanent method may be best. The current configuration of the camera should be addressed as well since the camera work envelop is not currently matched with the Powerball working envelope. Interfacing the gripper from ROS as well would also be of great interest to extend the functionality of the system all in the same framework. The Powerball arm has shown great potential for various applications at this early stage of set-up and experimentation and should be a productive testbed for further experimentation.

## REFERENCES

- [1] Powerball lightweight arm lwa 4p. [schunk.com/Powerball Lightweight Arm LWA 4P](http://schunk.com/Powerball%20Lightweight%20Arm%20LWA%204P), 2013.
- [2] Subir Kumar Saha. *Introduction to robotics*. Tata McGraw-Hill, 2008.
- [3] Robert J Schilling. *Fundamentals of robotics: analysis and control*. Simon & Schuster Trade, 1996.
- [4] B. Paden. *Kinematics and Control Robot Manipulators*. Phd thesis department of electrical engineering and computer science, University of California Berkley, 1986.
- [5] Milan Ikits and John M Hollerbach. Kinematic calibration using a plane constraint. In *Robotics and Automation, 1997. Proceedings., 1997 IEEE International Conference on*, volume 4, pages 3191–3196. IEEE, 1997.
- [6] John M Hollerbach and Charles W Wampler. The calibration index and taxonomy for robot kinematic calibration methods. *The international journal of robotics research*, 15(6):573–591, 1996.
- [7] John M Hollerbach, Lydia Giugovaz, Martin Buehler, and Yangming Xu. Screw axis measurement for kinematic calibration of the sarcos dextrous arm. In *Intelligent Robots and Systems' 93, IROS'93. Proceedings of the 1993 IEEE/RSJ International Conference on*, volume 3, pages 1617–1621. IEEE, 1993.
- [8] James Stewart. *Essential Calculus: Early Transcendentals*. CengageBrain. com, 2010.
- [9] Jorge Santolaria and Manuel GinéS. Uncertainty estimation in robot kinematic calibration. *Robotics and Computer-Integrated Manufacturing*, 2012.
- [10] Neil Tenenholz. Nonlinear model fitting using least squares. [harvard.edu/ntenenholz](http://harvard.edu/ntenenholz), 2012.
- [11] Projection of a point to a plane. [mathworks.com/plane\\_projection](http://mathworks.com/plane_projection), 2010.
- [12] Minimum distance between skew lines. [mathworks.com/line\\_distance](http://mathworks.com/line_distance), 2007.
- [13] Bruno Siciliano and Oussama Khatib. *Springer handbook of robotics*. Springer, 2008.

## A. INVERSE KINEMATICS MATLAB CODE

```

1 function th_out=powerball_inverse_kinematics(T06,th_p)
2 %%#codegen
3 %% Inverse Kinematics Schunk Powerball LWA 4.6
4 %       th_out=inverse_kinematics(T06, theta_previous)
5 % T06= [4x4]
6 % theta_previous= [1x6]
7
8
9 th_ik=zeros(7,8); % initialize working solution variable
10 th_limit=[170; 110; 155; 170; 140; 170;]*pi/180; %Symetric Joint limits
    Schunk Powerball 4.6
11 d_1=205;          %base
12 a_2=350;          %upper arm
13 d_4=305;          %forearm
14 d_6=75;           %hand
15
16
17
18 %%
19 % Solve for  $\theta_3$ 
20 dx=T06(1:3,1:3)*[0;0;d_6]; % Vector from
    spherical wrist to tool tip
21 d_elbow=T06(1:3,4)-dx-[0;0;d_1]; % Vector with the tool tip
    distance and base distance removed
22 d_elbow_norm =norm(d_elbow); % Distance from joints 1&2 to
    spherical wrist
23
24 temp=pi-acos((a_2^2+d_4^2-d_elbow_norm^2)/(2*a_2*d_4)); % Angle
    of Elbow
25
26 th_ik(3,1:4)=temp;          th_ik(7,1:4)= th_ik(7,1:4)+ 2^1;% Elbow up
27 th_ik(3,5:8)=-temp;      %th_ik(7,5:8)=th_ik(7,5:8)+0*2^1; % Elbow Down
28
29
30 %%
31 % Solve for  $\theta_1$  &  $\theta_2$ 
32
33 [theta1_1,theta2_1]=subproblem2([0;0;1],[0;1;0],[0; 0; a_2; ]+
    [-d_4*sin(th_ik(3,1)); 0; d_4*cos(th_ik(3,1))],d_elbow); % 2

```

```

    Solutions for theta3(1) Elbow up
34 [theta1_2, theta2_2]=subproblem2([0;0;1],[0;1;0],[0; 0; a_2;
    ]+[-d_4*sin(th_ik(3,5)); 0; d_4*cos(th_ik(3,5))],d_elbow); % 2
    Solutions for theta3(2) Elbow down
35
36 % Replace invalid solutions from subproblem2
37 if isnan(theta1_1(1)) || isnan(theta1_1(2))
38     theta1_1=[th_p(1), th_p(1)];
39 end
40
41 if isnan(theta1_2(1)) || isnan(theta1_2(2))
42     theta1_2=[th_p(1), th_p(1)];
43 end
44
45 if isnan(theta2_1(1)) || isnan(theta2_1(2))
46     theta2_1=[th_p(1), th_p(1)];
47 end
48 if isnan(theta2_2(1)) || isnan(theta2_2(2))
49     theta2_2=[th_p(1), th_p(1)];
50 end
51
52 th_ik(1,1:4)=[theta1_1(1:2) theta1_1(1:2)];
    th_ik(7,[1,3])=th_ik(7,[1,3])+2^0; %Shoulder right
53 th_ik(1,5:8)=[theta1_2(1:2) theta1_2(1:2)];
    th_ik(7,[5,7])=th_ik(7,[5,7])+2^0; %Shoulder right
54 th_ik(2,1:4)=[theta2_1(1:2) theta2_1(1:2)];
55 th_ik(2,5:8)=[theta2_2(1:2) theta2_2(1:2)];
56
57 %%
58 % $\theta_4$, $\theta_5$, $\theta_6$
59 for z=[1:2, 5:6]
60     th_1=th_ik(1,z);
61     th_2=th_ik(2,z);
62     th_3= th_ik(3,z);
63 %     [th_1,th_2,th_3]
64     T01 = symDH(th_1, d_1, 0, -pi/2); % Create
        homogenous transform for joint angle 1 for EACH joint
        configuration.
65     T12 = symDH(th_2-pi/2, 0, a_2, pi);
66     T23 = symDH(th_3-pi/2, 0, 0,-pi/2);
67 %
68

```

```

69     T02=T01*T12;
70     T03=T02*T23;           % Create homogenous transform for first 3
                             % joint angles for EACH joint configuration.
71
72     Twrist=([T03(1:3,1:3) .', -T03(1:3,1:3) .'*T03(1:3,4);0 0 0
              1])*T06;       % Remove first 3 joint angles
                             % to isolate joints  $\theta_4$ ,  $\theta_5$ ,  $\theta_6$ 
73
74     th_ik(4,z)=atan2(-Twrist(2,3),-Twrist(1,3));
                             th_ik(7,z)=th_ik(7,z)+2^2; %Wrist UP
75     th_ik(5,z)=acos(Twrist(3,3)); %
76     th_ik(6,z)=atan2(-Twrist(3,2),Twrist(3,1));
77
78     th_ik(4,z+2)=atan2(Twrist(2,3),Twrist(1,3));
                             %th_ik(7,z+2)=th_ik(7,z+2)+0; %Wrist DOWN
79     th_ik(5,z+2)=-acos(Twrist(3,3)); %
80     th_ik(6,z+2)=atan2(Twrist(3,2),-Twrist(3,1));
81 end
82
83 %%
84 % Joint limits and find closest to previous
85 counter=0;
86 temp_out=zeros(7,8);
87 for x=1:8                   %Joint limit check
88     if sum(abs(th_ik(1:6,x)) <=th_limit)==6
89         counter=counter+1;
90         temp_out(1:7,counter)=th_ik(1:7,x);
91     end
92 end
93     temp_out_lim=temp_out(1:7,1:counter); % Keep solutions
                             % within Joint limits
94
95 if max(abs(th_p))>0         % Check for existance of th_p
    (theta previous)
96     min_diff=zeros(counter,1);
97     for x=1:counter         %Find closest solution to th_p
    (theta previous)
98         min_diff(x)=norm(temp_out_lim(1:6,x) .'- th_p);           %
                             % Distance of each solution
99
100 end

```



```

101         th_out=temp_out_lim(:, min_diff==min(min_diff)); % Use closest
           solution
102     else
103         th_out=temp_out_lim; % Use all solutions with joint limits
104     end
105
106
107
108
109     end
110
111     function T = symDH(th,d,a,al)
112     % homogenous transform matrix from DH parameters
113     T=[   cos(th)           -sin(th)*cos(al)       sin(th)*sin(al)
          a*cos(th);...
114         sin(th)           cos(th)*cos(al)         -cos(th)*sin(al)
          a*sin(th);...
115         0                 cos(al)                sin(al)
          0                 0                      0
116         0                 0                      0
           1];
117     end
118
119     function [theta1,theta2]=subproblem2(k1,k2,p,q)
120     % Finds theta1 & theta2 where two vectors p & q intersect when rotated
           about vectors k1 and k2
121     k12=k1'*k2;
122     pk=p'*k2;
123     qk=q'*k1;
124
125     % check if solution exists
126
127     if abs(k12^2-1)<eps;theta1=[NaN NaN]; theta2=[NaN NaN];
128         %disp('no solution (**1**)' );
129         return;
130     end
131
132     a=[k12 -1;-1 k12]*[pk;qk]/(k12^2-1);
133
134     bb=(norm(p)^2-norm(a)^2-2*a(1)*a(2)*k12);
135     if abs(bb)<eps;bb=0;end

```

```

136 if bb<0;theta1=[NaN NaN];theta2=[NaN NaN];
137 %     disp('no solution (**2**) ');
138     return;
139 end
140
141 % check if there is only 1 solution
142 gamma=sqrt(bb)/norm(cross(k1,k2));
143 if abs(gamma)<eps;
144     c1=[k1 k2 cross(k1,k2)]*[a;gamma];
145     theta2=[subproblem1(k2,p,c1) NaN];
146     theta1=[-subproblem1(k1,q,c1) NaN];
147 %     disp('One solution ');
148     return
149 end
150
151 % general case: 2 solutions
152
153 theta1=zeros(1,2);
154 theta2=zeros(1,2);
155
156 c1=[k1 k2 cross(k1,k2)]*[a;gamma];
157 c2=[k1 k2 cross(k1,k2)]*[a;-gamma];
158 theta2(1)=subproblem1(k2,p,c1);
159 theta2(2)=subproblem1(k2,p,c2);
160
161 theta1(1)=-subproblem1(k1,q,c1);
162 theta1(2)=-subproblem1(k1,q,c2);
163 end
164
165 function [theta]=subproblem1(k,p,q)
166 % [theta]=subproblem1(k,p,q)
167 % 1 unique solution
168
169 k=k/norm(k);
170 pp=p-(p.'*k)*k;
171 qp=q-(q.'*k)*k;
172
173 epp=pp/norm(pp);
174 eqp=qp/norm(qp);
175
176 theta=subproblem0(epp,eqp,k);
177 end

```

```

178
179 function [theta]=subproblem0(p,q,k)
180 % [theta]=subproblem0(p,q,k)
181 % 1 unique solution
182
183
184 pp=p/norm(p);
185 qp=q/norm(q);
186
187 theta=2*atan2(norm(pp-qp),norm(pp+qp));
188
189 if k'*(cross(p,q))<0
190     theta=-theta;
191 end
192 end

```

## B. CIRCULAR POINT ANALYSIS MATLAB CODE

```

1 function powerball_kinematics_ID_cpa_ros()
2 close all
3 tic
4 %% Symbolic computation of fwd kin
5 % syms th_0 d_0 a_0 al_0; T00=symDH(th_0, d_0, a_0, al_0);
6 % th = sym('th_%d', [6 1]);
7 % th0 = sym('th0_%d', [6 1]);
8 % d = sym('d_%d', [6 1]);
9 % a = sym('a_%d', [6 1]);
10 % al = sym('al_%d', [6 1]);
11 % s_pi2 = sym(pi/2);
12 % T01 = symDH(th(1)+th0(1), d(1), a(1), al(1))
13 % T12 = symDH(th(2)+th0(2), d(2), a(2), al(2))
14 % T23 = symDH(th(2)+th0(3), d(3), a(3), al(3))
15 % T34 = symDH(th(2)+th0(4), d(4), a(4), al(4))
16 % T45 = symDH(th(5)+th0(5), d(5), a(5), al(5))
17 % T56 = symDH(th(6)+th0(6), d(6), a(6), al(6))
18 % T06 = (T01 * T12 * T23 * T34 * T45 * T56);
19 % T01=rewrite(T01, 'sincos');
20 % T02=rewrite(T01*T12, 'sincos');
21 % T03=rewrite(T02*T23, 'sincos');
22 % T04=rewrite(T03*T34, 'sincos');
23 % T05=rewrite(T04*T45, 'sincos');
24 % T06=rewrite(T05*T56, 'sincos');

```

```

25
26 % NOMINAL *****
27 % T01 = symDH(th(1), d(1), 0, s_pi2);
28 % T12 = symDH(th(2)+s_pi2, 0, a(2), 0);
29 % T23 = symDH(th(3)+s_pi2, 0, 0, s_pi2);
30 % T34 = symDH(th(4), d(4), 0, s_pi2);
31 % T45 = symDH(th(5)+pi, 0, 0, s_pi2);
32 % T56 = symDH(th(6), d(6), 0, 0);
33
34 %% Nominal values
35 d_1=.205;
36 a_2=.350;
37 d_4=.305;
38 d_6=.075;
39 m_d=400; m_a=3*pi/2; % Max distance, max angle
40 DH0=[0 d_1 0 pi/2; ...
41      pi/2 0 a_2 0; ...
42      pi/2 0 0 pi/2; ...
43      0, d_4, 0, pi/2;...
44      pi, 0, 0, pi/2;...
45      0, d_6, 0, 0];
46
47 pose_import
48
49 %% Extrapolation of orientation
50 % vr_mark=[0;0; .1];
51 % for i=1:length(time_p)
52 %     theta=2*atan2(norm([xq(i),yq(i),zq(i)]),w(i)));
53 %     vr_pos=expm(hat([xq(i) yq(i) zq(i)]))*theta)*vr_mark;
54 %     x2(i)=x(i)+vr_pos(1);
55 %     y2(i)=y(i)+vr_pos(2);
56 %     z2(i)=z(i)+vr_pos(3);
57 % end
58 % x=[x x2.'];
59 % y=[y y2.'];
60 % z=[z z2.'];
61
62
63 starts_t=(4.25+[1:10:6*10]); stops_t=(14.25+[1:10:6*10]);
64 for i=1:6
65     starts_1(i)=find(time_p<=starts_t(i),1,'last');
66     stops_1(i)=find(time_p<=stops_t(i),1,'last');

```

```

67 end
68
69
70 plane_normal_out1=(-plane_normal(x([starts_1(1):stops_1(1)
    ],1),y([starts_1(1):stops_1(1) ],1),z([starts_1(1):stops_1(1) ],1)
    ,1,1)).';
71 center_point_out1=(circle_point(plane_normal_out1,x([starts_1(1):stops_1(1)
    ],1),y([starts_1(1):stops_1(1) ],1),z([starts_1(1):stops_1(1) ],1)
    ,1,1)).';
72 z_cam=[0;0;-1];
73 n_cam=(hat(z_cam)*plane_normal_out1)/norm(cross(z_cam,plane_normal_out1))
    ;
74 theta_cam=acos(sum(z_cam.*plane_normal_out1));
75 R_cam=expm(hat(n_cam)*theta_cam).';
76 axis_offset=R_cam*center_point_out1;
77
78 plane_normal_out2=(-plane_normal(x([starts_1(2):stops_1(2)
    ],1),y([starts_1(2):stops_1(2) ],1),z([starts_1(2):stops_1(2) ],1)
    ,2,1)).';
79 center_point_out2=(circle_point(plane_normal_out1,x([starts_1(2):stops_1(2)
    ],1),y([starts_1(2):stops_1(2) ],1),z([starts_1(2):stops_1(2) ],1)
    ,2,1)).';
80
81
82
83 %% Change Coordinate system to align axis 1 with Z
84 axis_offset(3)= 0.0345275294433222-.205; % difference between nominal
    value and current distance between axes 1 & 2
85 for i=1:size(x,2)
86     temp_pos=R_cam*[x(:,i)';y(:,i)';z(:,i)'];
87     x(:,i)=(temp_pos(1,:)-axis_offset(1)).';
88     y(:,i)=(temp_pos(2,:)-axis_offset(2)).';
89     z(:,i)=(temp_pos(3,:)-axis_offset(3)).';
90 end
91 theta_1_offset=-atan2(y(starts_1(1)),x(starts_1(1))); % theta1 offset
92 R_theta_1_offset=expm(hat([0;0;1;]) *theta_1_offset); % theta1 offset
    rotation matrix
93
94 for i=1:size(x,2) % Align data with theta1 offset
95     temp_pos=R_theta_1_offset*[x(:,i)';y(:,i)';z(:,i)'];
96     x(:,i)=temp_pos(1,:).';
97     y(:,i)=temp_pos(2,:).';

```

```

98         z(:,i)=temp_pos(3,:).';
99     end
100
101
102     close all
103
104     %% Calculate each axis and angle
105     for j=1:size(x,2)           % # of
106         j                       % # of
107         for i=1:6
108
109             plane_normal_out=plane_normal(x([starts_1(i):stops_1(i)
110                 ],j),y([starts_1(i):stops_1(i)
111                 ],j),z([starts_1(i):stops_1(i) ],j) ,i,i);
112
113             center_point_out=circle_point(plane_normal_out,x([starts_1(i):stops_1(i)
114                 ],j),y([starts_1(i):stops_1(i)
115                 ],j),z([starts_1(i):stops_1(i) ],j) ,i,i);
116
117             %
118             center_point_out=R_cam*center_point_out.'+[axis_offset(1)
119                 axis_offset(2) 0].';
120
121             %
122             plane_normal_out=R_cam*plane_normal_out.'+[axis_offset(1)
123                 axis_offset(2) 0].';
124
125             joint_axes(i,1:6,j)=[center_point_out , plane_normal_out];
126
127         end
128     end
129
130     legend(['axis 1 data'},{ 'Z axis 1'},{ 'axis 2 data'},{ 'Z axis 2'},{ 'axis
131         3 data'},{ 'Z axis 3'},{ 'axis 4 data'},{ 'Z axis 4'},{ 'axis 5
132         data'},{ 'Z axis 5'},{ 'axis 6 data'},{ 'Z axis 6'}]);
133     joint_axes(:, :, 1)
134
135     %% distances between axes
136     line_style=['or'; 'sg'; '^b'; 'om'; 'sk'; '^c'];
137     for i=1:5
138         center_point_out1=joint_axes(i,1:3);
139         plane_normal_out1=joint_axes(i,4:6);

```

```

129     center_point_out2=joint_axes(i+1,1:3);
        plane_normal_out2=joint_axes(i+1,4:6);
130     n_zdist=cross(plane_normal_out2,plane_normal_out1);
        n_zdist=n_zdist/norm(n_zdist);
131     dist_vect=center_point_out2-center_point_out1;
132     dist_1=center_point_out1 -
        dot(cross(center_point_out2-center_point_out1,plane_normal_out2),n_zdist)/dot(n
133     dist_2=center_point_out2 -
        dot(cross(center_point_out2-center_point_out1,plane_normal_out1),n_zdist)/dot(n
134
135     norm(dist_2-dist_1)
136
137     joint_offsets(i,1:6)=[dist_1 dist_2];
138
139 %     plot3(dist_1(1),dist_1(2),dist_1(3),[ line_style(i,1)
        'k'],dist_2(1),dist_2(2),dist_2(3),[ line_style(i,1)
        'k'],'MarkerFaceColor',line_style(i,2), 'MarkerSize', 9)
140
141 end
142 i=3;
143     center_point_out1=joint_axes(3,1:3);
        plane_normal_out1=joint_axes(3,4:6);
144     center_point_out2=joint_axes(5,1:3);
        plane_normal_out2=joint_axes(5,4:6);
145     n_zdist=cross(plane_normal_out2,plane_normal_out1);
        n_zdist=n_zdist/norm(n_zdist);
146     dist_vect=center_point_out2-center_point_out1;
147     dist_1=center_point_out1 -
        dot(cross(center_point_out2-center_point_out1,plane_normal_out2),n_zdist)/dot(n
148     dist_2=center_point_out2 -
        dot(cross(center_point_out2-center_point_out1,plane_normal_out1),n_zdist)/dot(n
149
150     norm(dist_2-dist_1)
151     [dist_1 dist_2]
152
153 %     plot3(dist_1(1),dist_1(2),dist_1(3),[ line_style(i,1)
        'k'],dist_2(1),dist_2(2),dist_2(3),[ line_style(i,1)
        'k'],'MarkerFaceColor',line_style(i,2), 'MarkerSize', 9)
154
155
156 joint_offsets
157

```

```

158 figure(321); cla; hold on; plot3(x,y,z, 'b'); plot3( [0 1]*x(1),[0
      1]*y(1),[0 1]*z(1), '+-k', 'LineWidth',1.25); title('Track data'); grid
      on; view(3); axis equal;
159
160 figure(322); clf;
161 px(1)=subplot(3,1,1);
162 plot(time_p,x, 'r', 'LineWidth',2); grid on;
163 ylabel('x (m)', 'FontSize',font_size); title('End Effector Trajectory in
      Time', 'FontSize',font_size);
164
165 px(2)=subplot(3,1,2);
166 plot(time_p,y, 'g', 'LineWidth',2); grid on;
167 ylabel('y (m)', 'FontSize',font_size);
168
169 px(3)=subplot(3,1,3);
170 plot(time_p,z, 'b', 'LineWidth',2); grid on;
171 ylabel('z (m)', 'FontSize',font_size); xlabel('time
      (s)', 'FontSize',font_size);
172
173 linkaxes(px, 'x');
174
175 whos x y z time_p start_time_p
176 % save('pose_data_2013-12-07T18-42-12_aligned', 'x','y','z', 'time_p',
      'start_time_p')
177
178
179 toc
180
181 return
182
183
184 function T06_out=end_effector_error(DH_param)
185 %% ERROR function
186 % DH_param = []
187 % T06_in = [4x4xn]
188 % th = [6xn]
189 load('powerball_ID_gen.mat')
190 th=[th_1.' th_2.' th_3.' th_4.' th_5.' th_6.'];
191 T06_out=zeros(4,4,size(th,1));
192 for x=1:size(th,1)
193     T06_out(1:4,1:4,x)=end_effector(DH_param, th(x,:));
194

```



```

195         %         error_out(1:4,1:4,x)=(    T06_out-T06(1:4,1:4,x)    ).^2;
196
197     end
198
199     return
200
201     function T06_out=end_effector(DH_param, th)
202     %% End effector function
203
204     % DH_param=[]
205     % th = [6x1]
206
207     th_1=th(1);th_2=th(2);th_3=th(3);th_4=th(4); th_5=th(5);th_6=th(6);
208     [th0_1]=DH_param(1,1); [d_1]=DH_param(1,2); [a_1]=DH_param(1,3);
209     [al_1]=DH_param(1,4);
210     [th0_2]=DH_param(2,1);
211     [d_2]=DH_param(2,2);[a_2]=DH_param(2,3);[al_2]=DH_param(2,4);
212     [th0_3]=DH_param(3,1);
213     [d_3]=DH_param(3,2);[a_3]=DH_param(3,3);[al_3]=DH_param(3,4);
214     [th0_4]=DH_param(4,1);
215     [d_4]=DH_param(4,2);[a_4]=DH_param(4,3);[al_4]=DH_param(4,4);
216     [th0_5]=DH_param(5,1);
217     [d_5]=DH_param(5,2);[a_5]=DH_param(5,3);[al_5]=DH_param(5,4);
218     [th0_6]=DH_param(6,1);
219     [d_6]=DH_param(6,2);[a_6]=DH_param(6,3);[al_6]=DH_param(6,4);
220
221     %%
222     T01 =...
223     [ cos(th0_1 + th_1), -sin(th0_1 + th_1)*cos(al_1), sin(th0_1 +
224     th_1)*sin(al_1), a_1*cos(th0_1 + th_1);...
225     sin(th0_1 + th_1), cos(th0_1 + th_1)*cos(al_1), -cos(th0_1 +
226     th_1)*sin(al_1), a_1*sin(th0_1 + th_1);...
227     0, sin(al_1), cos(al_1),
228     d_1;...
229     0, 0, 0,
230     1];
231
232     T12 =....
233     [ cos(th0_2 + th_2), -sin(th0_2 + th_2)*cos(al_2), sin(th0_2 +
234     th_2)*sin(al_2), a_2*cos(th0_2 + th_2);...

```

```

225     sin(th0_2 + th_2),  cos(th0_2 + th_2)*cos(al_2), -cos(th0_2 +
      th_2)*sin(al_2), a_2*sin(th0_2 + th_2);...
226     0,                sin(al_2),                cos(al_2),
      d_2;...
227     0,                0,                0,
      1];
228
229
230 T23 =...
231     [ cos(th0_3 + th_3), -sin(th0_3 + th_3)*cos(al_3),  sin(th0_3 +
      th_3)*sin(al_3), a_3*cos(th0_3 + th_3) ; ...
232     sin(th0_3 + th_3),  cos(th0_3 + th_3)*cos(al_3), -cos(th0_3 +
      th_3)*sin(al_3), a_3*sin(th0_3 + th_3) ; ...
233     0,                sin(al_3),
      cos(al_3),
      d_3 ; ...
234     0,                0,
      0,
      1];
235
236
237 T34 =....
238     [ cos(th0_4 + th_4), -sin(th0_4 + th_4)*cos(al_4),  sin(th0_4 +
      th_4)*sin(al_4), a_4*cos(th0_4 + th_4) ; ...
239     sin(th0_4 + th_4),  cos(th0_4 + th_4)*cos(al_4), -cos(th0_4 +
      th_4)*sin(al_4), a_4*sin(th0_4 + th_4) ; ...
240     0,                sin(al_4),
      cos(al_4),
      d_4 ; ...
241     0,                0,
      0,
      1];
242
243
244 T45 =...
245     [ cos(th0_5 + th_5), -sin(th0_5 + th_5)*cos(al_5),  sin(th0_5 +
      th_5)*sin(al_5), a_5*cos(th0_5 + th_5);...
246     sin(th0_5 + th_5),  cos(th0_5 + th_5)*cos(al_5), -cos(th0_5 +
      th_5)*sin(al_5), a_5*sin(th0_5 + th_5);...
247     0,                sin(al_5),                cos(al_5),
      d_5;...

```

```

248         0,                                0,                                0,
                                                1];
249
250
251 T56 =...
252     [ cos(th0_6 + th_6), -sin(th0_6 + th_6)*cos(al_6), sin(th0_6 +
      th_6)*sin(al_6), a_6*cos(th0_6 + th_6);...
253     sin(th0_6 + th_6), cos(th0_6 + th_6)*cos(al_6), -cos(th0_6 +
      th_6)*sin(al_6), a_6*sin(th0_6 + th_6);...
254     0,                                sin(al_6),                                cos(al_6),
      d_6;...
255     0,                                0,                                0,
                                                1];
256
257 T06_out = (T01 * T12 * T23 * T34 * T45 * T56);
258
259
260 return
261
262
263 function T = symDH(th,d,a,al)
264 %% Homogeneous transform
265 T=[    cos(th)            -sin(th)*cos(al)        sin(th)*sin(al)
      a*cos(th);...
266     sin(th)            cos(th)*cos(al)        -cos(th)*sin(al)
      a*sin(th);...
267     0                                sin(al)
      cos(al)                                d;
268     0                                0
      0                                0
      1];
269 return
270
271 function khat = hat(k)
272 khat=[0 -k(3) k(2); k(3) 0 -k(1); -k(2) k(1) 0];
273 return
274
275 function center_point_out=circle_point(plane_normal_out,x,y,z,i,j)
276
277 if size(plane_normal_out,1)~=1; plane_normal_out=plane_normal_out.'; end
278 %- 2*z*z0 + x^2 - 2*x*x0 + x0^2 + y^2 - 2*y*y0 + y0^2 + z^2 + z0^2 - r^2
279 % for i=1:length(x)

```

```

280 %
281 % end
282 x=squeeze(x); y=squeeze(y); z=squeeze(z);
283 if size(x,2)>1; x=x.'; y=y.'; z=z.'; end
284 y(isnan(x))=[]; z(isnan(x))=[]; x(isnan(x))=[];
285 y(isnan(z))=[]; x(isnan(z))=[]; z(isnan(z))=[];
286 z(isnan(y))=[]; x(isnan(y))=[]; y(isnan(y))=[];
287
288 % m=length(x);
289 % P=[x,y,z];
290 plane_normal_out = plane_normal_out/norm(plane_normal_out); % <-- do this
    if N is not normalized
291 Q=[mean(x) mean(y) mean(z)];
292 N2 = plane_normal_out.'*plane_normal_out;
293 % P0 = P*(eye(3)-N2)+repmat(Q*N2,m,1);
294 % x=P0(:,1); y=P0(:,2); z=P0(:,3);
295
296 A=[2*x, 2*y, 2*z, ones(length(x),1)];
297 q=x.^2+y.^2+z.^2;
298 center_point_out=pinv(A)*q;
299
300 center_point_out = center_point_out(1:3).'*(eye(3)-N2)+Q*N2;
301
302 font_size=15;
303 line_style=['or'; 'sg'; '^b'; 'om'; 'sk'; '^c'];
304 plane_normal_out =plane_normal_out *.25;
305 % figure(300+i); hold on; plot3(x,y,z,['-' line_style(j,2)]); grid on;
    xlabel('X (m)', 'FontSize', font_size); ylabel('Y
    (m)', 'FontSize', font_size); zlabel('Z (m)', 'FontSize', font_size);
    title(['Circle Center & Normal: axis ' num2str(i)
    ], 'FontSize', font_size); hold on; %plot3(P0(:,1),P0(:,2),
    P0(:,3), 'or');
306 % plot3(center_point_out(1),center_point_out(2),center_point_out(3),['p'
    line_style(j,2)], [0 plane_normal_out(1)]+center_point_out(1), [0
    plane_normal_out(2)]+center_point_out(2), [0
    plane_normal_out(3)]+center_point_out(3), ['-p'
    line_style(j,2)], 'MarkerFaceColor', line_style(j,2), 'MarkerSize', 9);
    view(3); axis equal;
307
308 figure(400); hold on; ax=gca; plot3(ax,x,y,z,['-' line_style(j,2)]);
    grid on; xlabel('X (m)', 'FontSize', font_size); ylabel('Y
    (m)', 'FontSize', font_size); zlabel('Z (m)', 'FontSize', font_size);

```

```

    title(['Joint Z Axes and Data using CPA'], 'FontSize', font_size);
    hold on; %plot3(P0(:,1), P0(:,2), P0(:,3), 'or');
309 plot3([0 plane_normal_out(1)+center_point_out(1), [0
    plane_normal_out(2)+center_point_out(2), [0
    plane_normal_out(3)+center_point_out(3), ['-p'
    line_style(j,2)], 'MarkerFaceColor', line_style(j,2), 'MarkerSize', 9);
    view(3); axis equal;

310
311 return
312
313 function plane_normal_out=plane_normal(x,y,z,i,j)
314 x=squeeze(x); y=squeeze(y); z=squeeze(z);
315 if size(x,2)>1; x=x.'; y=y.'; z=z.'; end
316
317 y(isnan(x))=[]; z(isnan(x))=[]; x(isnan(x))=[];
318 y(isnan(z))=[]; x(isnan(z))=[]; z(isnan(z))=[];
319 z(isnan(y))=[]; x(isnan(y))=[]; y(isnan(y))=[];
320
321 center_point_out=[mean(x),mean(y),mean(z)];
322 min_xyz=[max(x)-min(x) max(y)-min(y) max(z)-min(z)];
323
324 min_range=find(min_xyz==min(min_xyz));
325
326 max_dir=.25;
327
328 switch min_range
329     case 1
330         nx=max_dir;
331         A=[(y - center_point_out(2)) (z - center_point_out(3))];
332         plane_normal_temp=pinv(A)*-(x - center_point_out(1));
333         plane_normal_out=[nx plane_normal_temp.'*nx];
334     case 2
335         ny=max_dir;
336         A=[(x - center_point_out(1)) (z - center_point_out(3))];
337         plane_normal_temp=pinv(A)*-(y - center_point_out(2));
338         plane_normal_out=[plane_normal_temp(1)*ny ny
            plane_normal_temp(2)*ny];
339     case 3
340         nz=max_dir;
341         A=[(x - center_point_out(1)) (y - center_point_out(2))];
342         plane_normal_temp=pinv(A)*-(z - center_point_out(3));
343         plane_normal_out=[plane_normal_temp.'*nz nz];

```

```

344 end
345
346 % (nx*(x - x0) + ny*(y - y0) )/nz== (z - z0)
347
348 line_style=['or'; 'sg'; '^b'; 'om'; 'sk'; '^c'];
349
350 plane_normal_out = plane_normal_out/norm(plane_normal_out); % <--
    normalize normal vector
351 plane_normal_out =plane_normal_out *max_dir;
352 %
353 % figure(300+i); hold on; plot3(x,y,z,['-' line_style(j,2)]); grid on;
    xlabel('X (m)'); ylabel('Y (m)'); zlabel('Z (m)'); title(['PLANE:
    axis ' num2str(i) ]); hold on;
354 % plot3([0 plane_normal_out(1)],[0 plane_normal_out(2)],[0
    plane_normal_out(3)],['-p'
    line_style(j,2)], 'MarkerFaceColor', line_style(j,2)); view(3); axis
    equal;
355 %
356 % figure(400); hold on; plot3(x,y,z,['-' line_style(j,2)]); grid on;
    xlabel('X (m)'); ylabel('Y (m)'); zlabel('Z (m)'); title(['PLANE:
    axis ' num2str(i) ]); hold on;
357 % plot3([0 plane_normal_out(1)],[0 plane_normal_out(2)],[0
    plane_normal_out(3)],['-p'
    line_style(j,2)], 'MarkerFaceColor', line_style(j,2)); view(3); axis
    equal;
358
359 plane_normal_out = plane_normal_out/norm(plane_normal_out); % <--
    normalize normal vector
360 return

```

### C. GAUSS-NEWTON MINIMIZATION MATLAB CODE

```

1 function powerball_kinematics_ID_gauss()
2 close all
3
4 %% Symbolic computation of fwd kin
5 % syms th_0 d_0 a_0 al_0; T00=symDH(th_0, d_0, a_0, al_0);
6 th = sym('th_%d', [6 1]);
7 th0 = sym('th0_%d', [6 1]);
8 d = sym('d_%d', [6 1]);
9 a = sym('a_%d', [6 1]);
10 al = sym('al_%d', [6 1]);

```

```

11 th=sym(th, 'positive');
12 th0=sym(th0, 'positive');
13 d=sym(d, 'positive');
14 a=sym(a, 'positive');
15 al=sym(al, 'positive');
16
17 % d_1=.205;
18 % d_2=0;
19
20 % d([1,2,3, 5])=[.205, 0, 0, 0];
21 % a([1 3:6])=[0 0 0 0 0];
22 % phi_sym=[th0; d([4,6]); a([2]); al]
23 phi_sym=[th0; d; a; al];
24
25 % a_2=*;
26 % d_4=*;
27 % d_6=*;
28
29 T01 = symDH(th(1)+th0(1), d(1), a(1), al(1))
30 T12 = symDH(th(2)+th0(2), d(2), a(2), al(2))
31 T23 = symDH(th(3)+th0(3), d(3), a(3), al(3))
32 T34 = symDH(th(4)+th0(4), d(4), a(4), al(4))
33 T45 = symDH(th(5)+th0(5), d(5), a(5), al(5))
34 T56 = symDH(th(6)+th0(6), d(6), a(6), al(6))
35 % T06 = (T01 * T12 * T23 * T34 * T45 * T56);
36 T01=rewrite(T01, 'sincos');
37 T02=rewrite(T01*T12, 'sincos');
38 T03=rewrite(T02*T23, 'sincos');
39 T04=rewrite(T03*T34, 'sincos');
40 T05=rewrite(T04*T45, 'sincos');
41 T06=rewrite(T05*T56, 'sincos');
42
43
44 % J=jacobian(T06(1:3,4),phi_sym);
45
46 for i=1:length(phi_sym) % Create DH parameter Jacobian
47     J(1:3,i)=[diff(T06(1:3,4),phi_sym(i))];
48 end
49
50
51
52 % NOMINAL *****

```

```

53 % T01 = symDH(th(1), d(1), 0, s_pi2);
54 % T12 = symDH(th(2)+s_pi2, 0, a(2), 0);
55 % T23 = symDH(th(3)+s_pi2, 0, 0, s_pi2);
56 % T34 = symDH(th(4), d(4), 0, s_pi2);
57 % T45 = symDH(th(5)+pi, 0, 0, s_pi2);
58 % T56 = symDH(th(6), d(6), 0, 0);
59
60 %% Initial guess
61 d_1=.205;
62 a_2=0.319145526760135; %0.350;
63 d_4= 0.299986258998911; %0.305;
64 d_6=0.213876304123098 +.04; %0.075;
65 % m_d=400; m_a=3*pi/2; % Max distance, max angle
66 DH0=[0          d_1          0          -pi/2; ...
67       -pi/2     0          a_2          pi; ...
68       -pi/2     0          0          -pi/2; ...
69       0,         d_4,      0,         pi/2;...
70       0,         0,        0,         0,      -pi/2;...
71       0,         d_6,      0.100240233652924 ,      0];
72
73 % DH0=ones(size(DH0)); % BAD Starting position
74 % DH0=zeros(size(DH0)); % BAD Starting position
75 % DH0=ones(size(DH0)); % BAD Starting position
76 % DH0=zeros(size(DH0)); % BAD Starting position
77 % DH0=ones(size(DH0)); % BAD Starting position
78 % DH0=zeros(size(DH0)); % BAD Starting position
79 %% Data import
80 joint_import; clear th_*d
81 load('pose_data_2013-12-07T18-42-12_aligned.mat')
82
83
84 %% Align data start time and stop times
85 start_times=[start_time_p, start_time_j];
86 start_time_idx=find(start_times==max(start_times));
87
88 % Start *****
89 if start_time_idx==1
90     time_j=time_j-(start_time_p-start_time_j);
91     th_1(time_j<0)=[]; th_2(time_j<0)=[]; th_3(time_j<0)=[];
92     th_4(time_j<0)=[]; th_5(time_j<0)=[]; th_6(time_j<0)=[];

```



```

        time_j(time_j<0)=[];
92 else
93     time_p=time_p-3; %(start_time_j-start_time_p); %Hand calculated
        because there time is a time mismatch between the ROS times
94     x(time_p<0)=[]; y(time_p<0)=[]; z(time_p<0)=[];
        time_p(time_p<0)=[];
95 end
96
97 % Stop *****
98 stop_times=[time_p(end), time_j(end)];
99 start_time_idx=find(stop_times==max(stop_times));
100 if start_time_idx==1
101     stop_time_idx=find(time_p<=time_j(end),1,'last');
102     time_p(stop_time_idx:end)=[]; x(stop_time_idx:end)=[];
        y(stop_time_idx:end)=[]; z(stop_time_idx:end)=[];
103 else
104     stop_time_idx=find(time_j<=time_p(end),1,'last');
105     time_j(stop_time_idx:end)=[]; th_1(stop_time_idx:end)=[];
        th_2(stop_time_idx:end)=[]; th_3(stop_time_idx:end)=[];
        th_4(stop_time_idx:end)=[]; th_5(stop_time_idx:end)=[];
        th_6(stop_time_idx:end)=[];
106 end
107
108 for i=1:length(time_p) % if pose data is shorter vectors will be
        minimum length; if joint data is minimum vectors will be maximum
        length
109     time_diff=abs(time_p(i)-time_j);
110     joint_index=find(time_diff==min(time_diff));
111     th_1n(i)=th_1(joint_index); th_2n(i)=th_2(joint_index);
        th_3n(i)=th_3(joint_index); th_4n(i)=th_4(joint_index);
        th_5n(i)=th_5(joint_index); th_6n(i)=th_6(joint_index);
112     time_jn(i)=time_j(joint_index);
113 end
114
115 th_1=th_1n.'; th_2=th_2n.'; th_3=th_3n.'; th_4=th_4n.'; th_5=th_5n.';
        th_6=th_6n.';
116 time_j=time_jn.'; clear th_*n time_jn
117
118 %% Throw out data with bad timing delays
119 bad_time_temp=abs(time_j-time_p);
120 bad_time_threshold=1e-4; %2e-4;

```

```

121 th_1 (bad_time_temp>bad_time_threshold) = [];
    th_2 (bad_time_temp>bad_time_threshold) = [];
    th_3 (bad_time_temp>bad_time_threshold) = [];
    th_4 (bad_time_temp>bad_time_threshold) = [];
    th_5 (bad_time_temp>bad_time_threshold) = [];
    th_6 (bad_time_temp>bad_time_threshold) = [];
    time_j (bad_time_temp>bad_time_threshold) = [];
    time_p (bad_time_temp>bad_time_threshold) = [];
    x (bad_time_temp>bad_time_threshold) = [];
    y (bad_time_temp>bad_time_threshold) = [];
    z (bad_time_temp>bad_time_threshold) = [];

122
123 bad_time_temp=(time_j<=2 | time_j>=61.5); % trim pre and post
124 th_1 (bad_time_temp) = []; th_2 (bad_time_temp) = []; th_3 (bad_time_temp) = [];
    th_4 (bad_time_temp) = []; th_5 (bad_time_temp) = [];
    th_6 (bad_time_temp) = []; time_j (bad_time_temp) = [];
    time_p (bad_time_temp) = []; x (bad_time_temp) = []; y (bad_time_temp) = [];
    z (bad_time_temp) = [];

125 %% Initial Data check
126 tic
127 for i=1:length(th_1)
128     T06_out=end_effector_mex(DH0, [th_1(i) th_2(i) th_3(i) th_4(i)
129         th_5(i) th_6(i)]);
130     x_check(i)=T06_out(1,4);
131     y_check(i)=T06_out(2,4);
132     z_check(i)=T06_out(3,4);
133
134 end
135
136 disp(['Data check: ' num2str(toc)]);
137 font_size=15;
138 figure(210); clf;
139 plot3(x_check,y_check,z_check,'r'); hold on;
140 grid on; %axis equal;
141 xlabel('x (m)', 'FontSize', font_size); ylabel('y
    (m)', 'FontSize', font_size); zlabel('z (m)', 'FontSize', font_size);
    title('3D View of End Effector Path', 'FontSize', font_size);

142
143 figure(212); clf;
144 px(1)=subplot(3,1,1);
145 plot(time_j,x_check,'om'); hold on;

```

```

146 % ylabel('x (m)', 'FontSize', font_size); title('Joint Based Trajectory VS
      Actual', 'FontSize', font_size);
147
148 px(2)=subplot(3,1,2);
149 plot(time_j, y_check, 'ok'); hold on;
150 % ylabel('y (m)', 'FontSize', font_size);
151
152 px(3)=subplot(3,1,3);
153 plot(time_j, z_check, 'oc'); hold on;
154 % ylabel('z (m)', 'FontSize', font_size); xlabel('time
      (s)', 'FontSize', font_size);
155
156 linkaxes(px, 'x');
157 toc
158
159 %% Plot results
160 font_size=15;
161 figure(210); %clf;
162 plot3(x,y,z, 'b', [0 0], [0 0], [0 1]* 1.04122069939116, 'pk');
163 axis equal; grid on;
164 xlabel('x (m)', 'FontSize', font_size); ylabel('y
      (m)', 'FontSize', font_size); zlabel('z (m)', 'FontSize', font_size);
      title('3D View of End Effector Path', 'FontSize', font_size);
165 legend([{'DH from CPA'}, {'Aligned Camera Points'}], 'Location',
      'BestOutside', 'FontSize', 10); legend('boxoff')
166
167 figure(212); %clf;
168 px(1)=subplot(3,1,1);
169 plot(time_p, x, 'r');
170 ylabel('x (m)', 'FontSize', font_size); title('Joint Based Trajectory VS
      Actual', 'FontSize', font_size);
171
172 px(2)=subplot(3,1,2);
173 plot(time_p, y, 'g');
174 ylabel('y (m)', 'FontSize', font_size);
175
176 px(3)=subplot(3,1,3);
177 plot(time_p, z, 'b');
178 ylabel('z (m)', 'FontSize', font_size); xlabel('time
      (s)', 'FontSize', font_size);
179
180 linkaxes(px, 'x');

```

```

181 set(gcf, 'Position' , [804 329 715 604]);
182
183 figure(200); clf;
184
185 ax(1)=subplot(6,1,1); plot(time_j,th_1, '.b'); title(['Joint Angles_{best
sync} #pts:' num2str(length(th_1))], 'FontSize', font_size);
ylabel('th_1 (rad)', 'FontSize', font_size);
186 ax(2)=subplot(6,1,2); plot(time_j,th_2, '.b'); ylabel('th_2
(rad)', 'FontSize', font_size);
187 ax(3)=subplot(6,1,3); plot(time_j,th_3, '.b'); ylabel('th_3
(rad)', 'FontSize', font_size);
188 ax(4)=subplot(6,1,4); plot(time_j,th_4, '.b'); ylabel('th_4
(rad)', 'FontSize', font_size);
189 ax(5)=subplot(6,1,5); plot(time_j,th_5, '.b'); ylabel('th_5
(rad)', 'FontSize', font_size);
190 ax(6)=subplot(6,1,6); plot(time_j,th_6, '.b'); ylabel('th_6
(rad)', 'FontSize', font_size); xlabel('time (s)', 'FontSize', font_size);
191
192 linkaxes([ax px], 'x');
193
194 % return
195
196 %% Solve DH
197 pose=[x;y;z];
198
199 A1=zeros(length(x),length(phi_sym));
200 A2=A1; A3=A1;
201
202 % for i=1:3; for j=1:length(phi_sym); J_text{i,j}=char(J(i,j)); end; end;
whos J_text
203 % fid = fopen('J.txt', 'w');
204 % for i=1:3; for j=1:length(phi_sym); fprintf(fid, ['J_out(' num2str(i)
', ' num2str(j) ')= %s ; \r\n'], J_text{i,j}); fprintf(fid, '\r\n');
end; end;
205 % fclose(fid);
206
207 whos
208 tic
209 i=0;
210 phi_change=ones(size(phi_sym)); % Initialize stopping criterion
211 phi_data(1:length(phi),1)=phi;
% Store Progress Initialize

```

```

212
213
214 pose_err_data(1:length(pose),1)=pose- [x_check.'; y_check.' ;
      z_check.']; % Store Progress
215 while max(abs(phi_change))>1e-4 && i<=60 %1e-4
216     i=i+1;
217     if rem(i,5)==0; disp(['Number of iterations: ' num2str(i) ',
      Change: ' num2str(max(abs(phi_change)))]); end
218
219     for j=1:length(x) % parfor Loop for each data point
220
221         T06_out=end_effector_mex(subs( [th0, d, a,
      al],phi_sym,phi), [th_1(j) th_2(j) th_3(j) th_4(j)
      th_5(j) th_6(j) ]); %subs(pose_dh,th, [th_1(j)
      th_2(j) th_3(j) th_4(j) th_5(j) th_6(j) ]);
222         pose1(j)=T06_out(1,4);
223         pose2(j)=T06_out(2,4);
224         pose3(j)=T06_out(3,4);
225
226         J_temp=J_eval_mex(subs( [th0, d, a, al],phi_sym,phi),
      [th_1(j) th_2(j) th_3(j) th_4(j) th_5(j) th_6(j) ]);
227         A1(j,:)=J_temp(1,:);
228         A2(j,:)=J_temp(2,:);
229         A3(j,:)=J_temp(3,:);
230
231
232     end
233     A=[A1;A2;A3];
234     pose_calc=[pose1.';pose2.';pose3.'];
235     pose_err=pose-pose_calc;
236
237     phi_delta=pinv(A)*(pose_err); % Solve least squares
      problem
238     phi=phi+phi_delta*2/(1+5^.2);
239     phi([1:6,19:24])=mod(phi([1:6,19:24]), 2*pi); %
      keep angles +/- 2*pi
240 % phi(8:9)=0;
      % Account for parallel adjacent axes instead of Hayati (not
      necessary)
241
242     phi_data(1:length(phi),i+1)=phi;
      % Store Progress

```

```

243     pose_err_data(1:length(pose),i+1)=pose_err;
           % Store Progress
244
245     phi_change=phi_data(:,i)-phi;
246
247 end% 1e-5
248 disp(['Total Number of iterations: ' num2str(i)]);
249
250 save('phi_data','phi_data','pose_err_data','th_1','th_2','th_3',
      'th_4','th_5','th_6','x','y','z','time_j','time_p');
251
252 phi_plot           % Final Plotting
253
254
255 toc
256
257 return
258
259 function T = symDH(th,d,a,al)
260 %% Homogeneous transform
261 T=[   cos(th)           -sin(th).*cos(al)           sin(th).*sin(al)
      a.*cos(th);...
262   sin(th)           cos(th).*cos(al)
      -cos(th).*sin(al)           a.*sin(th);...
263   0                   sin(al)
           cos(al)                   d;
264   0                   0
           0
           1];
265 return

```

#### D. JACOBIAN FOR DH PARAMETERS MATLAB CODE

```

1 function J_out=J_eval(DH_param, th) %#codegen
2 %% Jacobian Function for End Effector Position, x,y,z \wrt all 24 DH
   Parameters
3
4 J_out=zeros(3,24);
5
6 th_1=th(1);th_2=th(2);th_3=th(3);th_4=th(4); th_5=th(5);th_6=th(6);
7 [th0_1]=DH_param(1,1); [d_1]=DH_param(1,2); [a_1]=DH_param(1,3);
   [al_1]=DH_param(1,4);

```

```

8 [ th0_2]=DH_param(2,1);
   [d_2]=DH_param(2,2);[a_2]=DH_param(2,3);[al_2]=DH_param(2,4);
9 [ th0_3]=DH_param(3,1);
   [d_3]=DH_param(3,2);[a_3]=DH_param(3,3);[al_3]=DH_param(3,4);
10 [ th0_4]=DH_param(4,1);
    [d_4]=DH_param(4,2);[a_4]=DH_param(4,3);[al_4]=DH_param(4,4);
11 [ th0_5]=DH_param(5,1);
    [d_5]=DH_param(5,2);[a_5]=DH_param(5,3);[al_5]=DH_param(5,4);
12 [ th0_6]=DH_param(6,1);
    [d_6]=DH_param(6,2);[a_6]=DH_param(6,3);[al_6]=DH_param(6,4);
13
14 J_out(1,1)= d_6*(cos(th0_5 + th_5)*sin(al_5)*(sin(al_4)*(sin(th0_3 +
   th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
   th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
   th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
15 th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
   th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
   th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
   th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
   th_2)*cos(al_1)*cos(al_2))) +
16 sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 + th_3)*(sin(th0_1 +
   th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
   th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
   th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
   th_2)*sin(th0_1 + th_1) +
17 cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))) - cos(th0_4 +
   th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
   sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
   th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +
18 th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
   th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
   th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
   cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
19 th_2)*cos(al_1)*cos(al_2))) - cos(al_5)*(cos(al_4)*(sin(th0_3 +
   th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
   th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
   th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
20 th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
   th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
   th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
   th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
   th_2)*cos(al_1)*cos(al_2))) -

```













$$\begin{aligned}
& d_4 * (\cos(al_3) * (\cos(th0_1 + th_1) * \cos(th0_2 + th_2) * \sin(al_2) - \\
& \sin(th0_1 + th_1) * \sin(th0_2 + th_2) * \cos(al_1) * \sin(al_2)) + \\
72 \quad & \cos(th0_3 + th_3) * \sin(al_3) * (\cos(th0_1 + th_1) * \cos(th0_2 + \\
& th_2) * \cos(al_2) - \sin(th0_1 + th_1) * \sin(th0_2 + \\
& th_2) * \cos(al_1) * \cos(al_2)) - \sin(th0_3 + th_3) * \sin(al_3) * (\cos(th0_1 + \\
& th_1) * \sin(th0_2 + th_2) + \cos(th0_2 + th_2) * \sin(th0_1 + \\
& th_1) * \cos(al_1))) - \\
73 \quad & d_6 * (\cos(al_5) * (\sin(th0_4 + th_4) * \sin(al_4) * (\sin(th0_3 + th_3) * (\cos(th0_1 \\
& + th_1) * \cos(th0_2 + th_2) * \cos(al_2) - \sin(th0_1 + th_1) * \sin(th0_2 + \\
& th_2) * \cos(al_1) * \cos(al_2)) + \cos(th0_3 + th_3) * (\cos(th0_1 + \\
& th_1) * \sin(th0_2 + th_2) + \cos(th0_2 + th_2) * \sin(th0_1 \\
74 \quad & + th_1) * \cos(al_1))) - \cos(al_4) * (\cos(al_3) * (\cos(th0_1 + th_1) * \cos(th0_2 + \\
& th_2) * \sin(al_2) - \sin(th0_1 + th_1) * \sin(th0_2 + \\
& th_2) * \cos(al_1) * \sin(al_2)) + \cos(th0_3 + th_3) * \sin(al_3) * (\cos(th0_1 + \\
& th_1) * \cos(th0_2 + th_2) * \cos(al_2) - \sin(th0_1 + th_1) * \sin(th0_2 \\
75 \quad & + th_2) * \cos(al_1) * \cos(al_2)) - \sin(th0_3 + th_3) * \sin(al_3) * (\cos(th0_1 + \\
& th_1) * \sin(th0_2 + th_2) + \cos(th0_2 + th_2) * \sin(th0_1 + \\
& th_1) * \cos(al_1))) + \cos(th0_4 + th_4) * \sin(al_4) * (\sin(al_3) * (\cos(th0_1 \\
& + th_1) * \cos(th0_2 + th_2) * \sin(al_2) - \sin(th0_1 + \\
76 \quad & th_1) * \sin(th0_2 + th_2) * \cos(al_1) * \sin(al_2)) - \cos(th0_3 + \\
& th_3) * \cos(al_3) * (\cos(th0_1 + th_1) * \cos(th0_2 + th_2) * \cos(al_2) - \\
& \sin(th0_1 + th_1) * \sin(th0_2 + th_2) * \cos(al_1) * \cos(al_2)) + \sin(th0_3 \\
& + th_3) * \cos(al_3) * (\cos(th0_1 + th_1) * \sin(th0_2 + th_2) + \\
77 \quad & \cos(th0_2 + th_2) * \sin(th0_1 + th_1) * \cos(al_1))) + \sin(th0_5 + \\
& th_5) * \sin(al_5) * (\cos(th0_4 + th_4) * (\sin(th0_3 + th_3) * (\cos(th0_1 + \\
& th_1) * \cos(th0_2 + th_2) * \cos(al_2) - \sin(th0_1 + th_1) * \sin(th0_2 + \\
& th_2) * \cos(al_1) * \cos(al_2)) + \cos(th0_3 + th_3) * (\cos(th0_1 + \\
78 \quad & th_1) * \sin(th0_2 + th_2) + \cos(th0_2 + th_2) * \sin(th0_1 + th_1) * \cos(al_1))) \\
& - \sin(th0_4 + th_4) * (\sin(al_3) * (\cos(th0_1 + th_1) * \cos(th0_2 + \\
& th_2) * \sin(al_2) - \sin(th0_1 + th_1) * \sin(th0_2 + \\
& th_2) * \cos(al_1) * \sin(al_2)) - \cos(th0_3 + th_3) * \cos(al_3) * (\cos(th0_1 + \\
79 \quad & th_1) * \cos(th0_2 + th_2) * \cos(al_2) - \sin(th0_1 + th_1) * \sin(th0_2 + \\
& th_2) * \cos(al_1) * \cos(al_2)) + \sin(th0_3 + th_3) * \cos(al_3) * (\cos(th0_1 + \\
& th_1) * \sin(th0_2 + th_2) + \cos(th0_2 + th_2) * \sin(th0_1 + \\
& th_1) * \cos(al_1))) + \cos(th0_5 + \\
80 \quad & th_5) * \sin(al_5) * (\sin(al_4) * (\cos(al_3) * (\cos(th0_1 + th_1) * \cos(th0_2 + \\
& th_2) * \sin(al_2) - \sin(th0_1 + th_1) * \sin(th0_2 + \\
& th_2) * \cos(al_1) * \sin(al_2)) + \cos(th0_3 + th_3) * \sin(al_3) * (\cos(th0_1 + \\
& th_1) * \cos(th0_2 + th_2) * \cos(al_2) - \sin(th0_1 + th_1) * \sin(th0_2 + \\
81 \quad & th_2) * \cos(al_1) * \cos(al_2)) - \sin(th0_3 + th_3) * \sin(al_3) * (\cos(th0_1 + \\
& th_1) * \sin(th0_2 + th_2) + \cos(th0_2 + th_2) * \sin(th0_1 + \\
& th_1) * \cos(al_1))) + \sin(th0_4 + th_4) * \cos(al_4) * (\sin(th0_3 +
\end{aligned}$$









```

112 th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2)*sin(al_2)
    - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)*sin(al_2)) -
    cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 +
113 th_2)*cos(al_1)*cos(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
    th_1)*sin(th0_2 + th_2) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1))) - a_3*sin(th0_3 + th_3)*(cos(th0_1 +
114 th_2)*cos(al_1)*cos(al_2)) - a_4*cos(th0_4 + th_4)*(sin(th0_3 +
    th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)*cos(al_2)) + cos(th0_3 +
115 th_2)*sin(th0_1 + th_1)*cos(al_1))) - a_5*cos(th0_5 + th_5)*(cos(th0_4 +
    th_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) + cos(th0_3 + th_3)*(cos(th0_1 +
    th_1)*sin(th0_2 +
116 th_2) + cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1))) - sin(th0_4 +
    th_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2)*sin(al_2) -
    sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)*sin(al_2)) - cos(th0_3
    + th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 +
117 th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
    th_1)*sin(th0_2 + th_2) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1))) - a_2*cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1) ;
118
119 J_out(1,3)= d_4*(cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2
    + th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3
    + th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
120 cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) -
    d_6*(sin(th0_5 + th_5)*sin(al_5)*(cos(th0_4 + th_4)*(cos(th0_3 +
    th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
    th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
121 th_1)*cos(al_1)*cos(al_2)) + sin(th0_3 + th_3)*(cos(th0_1 +
    th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1))) + sin(th0_4 + th_4)*(cos(th0_3 +
    th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
    th_1)*sin(th0_2 +
122 th_2)*cos(al_1)) - sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +

```





```

th_3)*cos(al_3)*(cos(th0_1 +
142 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))
- sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)))) +
143 cos(th0_5 + th_5)*cos(al_5)*(sin(al_4)*(cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 +
144 th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*cos(al_4)*(cos(th0_3
+ th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
145 th_1)*cos(al_1)*cos(al_2)) + sin(th0_3 + th_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1))) - cos(th0_4 + th_4)*cos(al_4)*(cos(th0_3 +
th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
146 th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3 +
th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)))) + a_6*cos(th0_6 + th_6)*(sin(th0_5 +
147 th_5)*(sin(al_4)*(cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) +
148 cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) + sin(th0_4 +
th_4)*cos(al_4)*(cos(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) +
149 sin(th0_3 + th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1))) - cos(th0_4 +
th_4)*cos(al_4)*(cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) -
150 sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) - cos(th0_5 +
th_5)*(cos(th0_4 + th_4)*(cos(th0_3 + th_3)*(cos(th0_1 +
151 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
+ cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) +

```

```

sin(th0_3 + th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1))) + sin(th0_4 +
152 th_4)*(cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) -
sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3 +
th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
153 th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)))) + a_5*sin(th0_5 +
th_5)*(sin(al_4)*(cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
154 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
+ cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) +
sin(th0_4 + th_4)*cos(al_4)*(cos(th0_3 + th_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
155 th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) + sin(th0_3 + th_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1))) - cos(th0_4 + th_4)*cos(al_4)*(cos(th0_3 +
156 th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3 +
th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
157 th_1)*cos(al_1)*cos(al_2))) - a_4*cos(th0_4 + th_4)*(cos(th0_3 +
th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) + sin(th0_3 + th_3)*(cos(th0_1 +
158 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)))
- a_5*cos(th0_5 + th_5)*(cos(th0_4 + th_4)*(cos(th0_3 +
th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
159 th_1)*cos(al_1)*cos(al_2)) + sin(th0_3 + th_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1))) + sin(th0_4 + th_4)*(cos(th0_3 +
th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 +
160 th_2)*cos(al_1)) - sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)))) ;
161
162 J_out(1,4)= a_6*cos(th0_6 + th_6)*(sin(th0_5 + th_5)*(cos(th0_4 +
th_4)*cos(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +

```



172  $th_4) * \cos(al_4) * (\sin(th0_3 + th_3) * (\cos(th0_1 + th_1) * \sin(th0_2 +$   
 $th_2) * \cos(al_2) - \sin(th0_1 + th_1) * \sin(al_1) * \sin(al_2) + \cos(th0_2 +$   
 $th_2) * \sin(th0_1 + th_1) * \cos(al_1) * \cos(al_2)) - \cos(th0_3 +$   
 $th_3) * (\cos(th0_1 + th_1) * \cos(th0_2 + th_2) - \sin(th0_1 +$   
173  $th_1) * \sin(th0_2 + th_2) * \cos(al_1))) + \sin(th0_4 +$   
 $th_4) * \cos(al_4) * (\sin(th0_3 + th_3) * \cos(al_3) * (\cos(th0_1 +$   
 $th_1) * \cos(th0_2 + th_2) - \sin(th0_1 + th_1) * \sin(th0_2 +$   
 $th_2) * \cos(al_1)) - \sin(al_3) * (\cos(th0_1 + th_1) * \sin(th0_2 +$   
 $th_2) * \sin(al_2) + \sin(th0_1 +$   
174  $th_1) * \cos(al_2) * \sin(al_1) + \cos(th0_2 + th_2) * \sin(th0_1 +$   
 $th_1) * \cos(al_1) * \sin(al_2)) + \cos(th0_3 + th_3) * \cos(al_3) * (\cos(th0_1 +$   
 $th_1) * \sin(th0_2 + th_2) * \cos(al_2) - \sin(th0_1 +$   
 $th_1) * \sin(al_1) * \sin(al_2) + \cos(th0_2 + th_2) * \sin(th0_1 +$   
175  $th_1) * \cos(al_1) * \cos(al_2))) + \sin(th0_5 + th_5) * \sin(al_5) * (\cos(th0_4 +$   
 $th_4) * (\sin(th0_3 + th_3) * \cos(al_3) * (\cos(th0_1 + th_1) * \cos(th0_2 +$   
 $th_2) - \sin(th0_1 + th_1) * \sin(th0_2 + th_2) * \cos(al_1)) -$   
 $\sin(al_3) * (\cos(th0_1 + th_1) * \sin(th0_2 + th_2) * \sin(al_2) +$   
176  $\sin(th0_1 + th_1) * \cos(al_2) * \sin(al_1) + \cos(th0_2 + th_2) * \sin(th0_1 +$   
 $th_1) * \cos(al_1) * \sin(al_2)) + \cos(th0_3 + th_3) * \cos(al_3) * (\cos(th0_1 +$   
 $th_1) * \sin(th0_2 + th_2) * \cos(al_2) - \sin(th0_1 +$   
 $th_1) * \sin(al_1) * \sin(al_2) + \cos(th0_2 + th_2) * \sin(th0_1 +$   
177  $th_1) * \cos(al_1) * \cos(al_2))) - \sin(th0_4 + th_4) * (\sin(th0_3 +$   
 $th_3) * (\cos(th0_1 + th_1) * \sin(th0_2 + th_2) * \cos(al_2) - \sin(th0_1 +$   
 $th_1) * \sin(al_1) * \sin(al_2) + \cos(th0_2 + th_2) * \sin(th0_1 +$   
 $th_1) * \cos(al_1) * \cos(al_2)) - \cos(th0_3 + th_3) * (\cos(th0_1 +$   
178  $th_1) * \cos(th0_2 + th_2) - \sin(th0_1 + th_1) * \sin(th0_2 +$   
 $th_2) * \cos(al_1))) - d_5 * (\cos(th0_4 + th_4) * \sin(al_4) * (\sin(th0_3 +$   
 $th_3) * (\cos(th0_1 + th_1) * \sin(th0_2 + th_2) * \cos(al_2) - \sin(th0_1 +$   
 $th_1) * \sin(al_1) * \sin(al_2) + \cos(th0_2 + th_2) * \sin(th0_1 +$   
179  $th_1) * \cos(al_1) * \cos(al_2)) - \cos(th0_3 + th_3) * (\cos(th0_1 +$   
 $th_1) * \cos(th0_2 + th_2) - \sin(th0_1 + th_1) * \sin(th0_2 +$   
 $th_2) * \cos(al_1))) + \sin(th0_4 + th_4) * \sin(al_4) * (\sin(th0_3 +$   
 $th_3) * \cos(al_3) * (\cos(th0_1 + th_1) * \cos(th0_2 + th_2) - \sin(th0_1 +$   
180  $th_1) * \sin(th0_2 + th_2) * \cos(al_1)) - \sin(al_3) * (\cos(th0_1 +$   
 $th_1) * \sin(th0_2 + th_2) * \sin(al_2) + \sin(th0_1 +$   
 $th_1) * \cos(al_2) * \sin(al_1) + \cos(th0_2 + th_2) * \sin(th0_1 +$   
 $th_1) * \cos(al_1) * \sin(al_2)) + \cos(th0_3 + th_3) * \cos(al_3) * (\cos(th0_1 +$   
 $th_1) * \sin(th0_2 +$   
181  $th_2) * \cos(al_2) - \sin(th0_1 + th_1) * \sin(al_1) * \sin(al_2) + \cos(th0_2 +$   
 $th_2) * \sin(th0_1 + th_1) * \cos(al_1) * \cos(al_2))) - a_5 * \cos(th0_5 +$   
 $th_5) * (\cos(th0_4 + th_4) * (\sin(th0_3 + th_3) * \cos(al_3) * (\cos(th0_1 +$   
 $th_1) * \cos(th0_2 + th_2) - \sin(th0_1 + th_1) * \sin(th0_2$

182 + th\_2)\*cos(al\_1)) - sin(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 +  
th\_2)\*sin(al\_2) + sin(th0\_1 + th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 +  
th\_2)\*sin(th0\_1 + th\_1)\*cos(al\_1)\*sin(al\_2)) + cos(th0\_3 +  
th\_3)\*cos(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_2) -  
183 sin(th0\_1 + th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
th\_1)\*cos(al\_1)\*cos(al\_2))) - sin(th0\_4 + th\_4)\*(sin(th0\_3 +  
th\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_2) - sin(th0\_1 +  
th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
184 th\_1)\*cos(al\_1)\*cos(al\_2)) - cos(th0\_3 + th\_3)\*(cos(th0\_1 +  
th\_1)\*cos(th0\_2 + th\_2) - sin(th0\_1 + th\_1)\*sin(th0\_2 +  
th\_2)\*cos(al\_1))) + a\_6\*sin(th0\_6 + th\_6)\*(cos(th0\_5 +  
th\_5)\*cos(al\_5)\*(cos(th0\_4 + th\_4)\*cos(al\_4)\*(sin(th0\_3 +  
th\_3)\*(cos(th0\_1 +  
185 th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_2) - sin(th0\_1 + th\_1)\*sin(al\_1)\*sin(al\_2)  
+ cos(th0\_2 + th\_2)\*sin(th0\_1 + th\_1)\*cos(al\_1)\*cos(al\_2)) -  
cos(th0\_3 + th\_3)\*(cos(th0\_1 + th\_1)\*cos(th0\_2 + th\_2) - sin(th0\_1 +  
th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_1))) + sin(th0\_4 +  
186 th\_4)\*cos(al\_4)\*(sin(th0\_3 + th\_3)\*cos(al\_3)\*(cos(th0\_1 + th\_1)\*cos(th0\_2  
+ th\_2) - sin(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_1)) -  
sin(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) + sin(th0\_1  
+ th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 +  
187 th\_2)\*sin(th0\_1 + th\_1)\*cos(al\_1)\*sin(al\_2)) + cos(th0\_3 +  
th\_3)\*cos(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_2) -  
sin(th0\_1 + th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
th\_1)\*cos(al\_1)\*cos(al\_2))) - sin(al\_5)\*(cos(th0\_4 +  
188 th\_4)\*sin(al\_4)\*(sin(th0\_3 + th\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 +  
th\_2)\*cos(al\_2) - sin(th0\_1 + th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 +  
th\_2)\*sin(th0\_1 + th\_1)\*cos(al\_1)\*cos(al\_2)) - cos(th0\_3 +  
th\_3)\*(cos(th0\_1 + th\_1)\*cos(th0\_2 + th\_2) - sin(th0\_1 +  
189 th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_1))) + sin(th0\_4 +  
th\_4)\*sin(al\_4)\*(sin(th0\_3 + th\_3)\*cos(al\_3)\*(cos(th0\_1 +  
th\_1)\*cos(th0\_2 + th\_2) - sin(th0\_1 + th\_1)\*sin(th0\_2 +  
th\_2)\*cos(al\_1)) - sin(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 +  
th\_2)\*sin(al\_2) + sin(th0\_1 +  
190 th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
th\_1)\*cos(al\_1)\*sin(al\_2)) + cos(th0\_3 + th\_3)\*cos(al\_3)\*(cos(th0\_1 +  
th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_2) - sin(th0\_1 +  
th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
191 th\_1)\*cos(al\_1)\*cos(al\_2))) + sin(th0\_5 + th\_5)\*cos(al\_5)\*(cos(th0\_4 +  
th\_4)\*(sin(th0\_3 + th\_3)\*cos(al\_3)\*(cos(th0\_1 + th\_1)\*cos(th0\_2 +  
th\_2) - sin(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_1)) -  
sin(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) +



```

192 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
    th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
193 th_1)*cos(al_1)*cos(al_2))) - sin(th0_4 + th_4)*(sin(th0_3 +
    th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
    th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
194 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)))) - a_4*cos(th0_4 + th_4)*(sin(th0_3 +
    th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
    th_1)*sin(th0_2
195 + th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
    th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 +
    th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
    sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
196 th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) + a_4*sin(th0_4 +
    th_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
    th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
197 th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2
    + th_2)*cos(al_1))) + a_5*sin(th0_5 + th_5)*(cos(th0_4 +
    th_4)*cos(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) +
198 cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
    th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1))) + sin(th0_4 +
    th_4)*cos(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
    th_1)*cos(th0_2 +
199 th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) -
    sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1
    + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
200 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
    + cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)))) ;
201
202 J_out(1,5)= d_6*(sin(th0_5 +
    th_5)*sin(al_5)*(sin(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
    th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
    th_3)*sin(al_3)*(cos(th0_1 +

```







```

th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
233 th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1)
+ cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) +
cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) +
234 cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) + cos(th0_4 +
th_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3
235 + th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)))));
236
237 J_out(1,6)= a_6*cos(th0_6 +
th_6)*(sin(al_5)*(cos(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 +
238 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))
+ cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) -
239 sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) -
240 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
th_4)*sin(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) +
241 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
242 th_1)*cos(al_1)*cos(al_2)))) + cos(th0_5 +
th_5)*cos(al_5)*(sin(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
243 th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -

```



```

th_1)*cos(th0_2 + th_2) -
254 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
255 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
+ cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)))) -
cos(th0_5 + th_5)*(sin(th0_4 + th_4)*(sin(th0_3 +
th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
256 th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 +
257 th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) + cos(th0_4 +
th_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
258 th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)))));
259
260 J_out(1,7)= 0 ;
261
262 J_out(1,8)= sin(th0_1 + th_1)*sin(al_1) ;
263
264 J_out(1,9)= cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2) ;
265
266 J_out(1,10)= cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +
sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 +
267 th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) ;
268
269 J_out(1,11)= cos(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +

```

```

th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) -
270 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) - sin(th0_4 +
271 th_4)*sin(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
272 th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
th_4)*sin(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 +
273 th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
274 th_1)*cos(al_1)*cos(al_2))) ;
275
276 J_out(1,12)= cos(al_5)*(cos(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2
+ th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2
+ th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 +
277 th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) - sin(th0_4 +
278 th_4)*sin(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
279 th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
th_4)*sin(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 +
280 th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +

```



```

281 th_1)*cos(al_1)*cos(al_2)))) - cos(th0_5 +
      th_5)*sin(al_5)*(sin(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
      th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
282 th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
      th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
      sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
283 th_1)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
      th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
      th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
      th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
284 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)))
      - cos(th0_4 + th_4)*cos(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1
      + th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
285 th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1)
      + cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) +
      cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) +
286 cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)))) - sin(th0_5 +
      th_5)*sin(al_5)*(sin(th0_4 + th_4)*(sin(th0_3 +
      th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
287 th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1)
      + cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) +
      cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) +
288 cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) + cos(th0_4 +
      th_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3
289 + th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_1)))) ;
290
291 J_out(1,13)= cos(th0_1 + th_1) ;
292
293 J_out(1,14)= cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_1) ;
294
295 J_out(1,15)= cos(th0_3 + th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) -
      sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3 +

```

```

th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
296 th_1)*cos(al_1)*cos(al_2)) ;
297
298 J_out(1,16)= - sin(th0_4 + th_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1
+ th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) +
299 cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) - cos(th0_4 +
300 th_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 +
301 th_1)*sin(th0_2 + th_2)*cos(al_1))) ;
302
303 J_out(1,17)= sin(th0_5 + th_5)*(sin(al_4)*(cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
304 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))
+ cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) +
305 sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) -
306 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))) - cos(th0_4 +
th_4)*cos(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) +
307 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
308 th_1)*cos(al_1)*cos(al_2)))) - cos(th0_5 + th_5)*(sin(th0_4 +
th_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 +

```

```

th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) -
sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
309 th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
310 th_1)*cos(al_1)*cos(al_2))) + cos(th0_4 + th_4)*(sin(th0_3 +
th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
311 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)))) ;
312
313 J_out(1,18)= sin(th0_6 +
th_6)*(sin(al_5)*(cos(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 +
314 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))
+ cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) -
315 sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) -
316 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
th_4)*sin(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) +
317 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
318 th_1)*cos(al_1)*cos(al_2)))) + cos(th0_5 +
th_5)*cos(al_5)*(sin(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
319 th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +

```



```

th_1)*cos(th0_2 + th_2) -
330 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
331 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
+ cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)))) -
cos(th0_5 + th_5)*(sin(th0_4 + th_4)*(sin(th0_3 +
th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
332 th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 +
333 th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) + cos(th0_4 +
th_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
334 th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)))));
335
336 J_out(1,19)= d_3*(sin(th0_1 + th_1)*cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(th0_1 + th_1)*sin(al_1)*sin(al_2)) +
d_4*(cos(al_3)*(sin(th0_1 + th_1)*cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(th0_1 + th_1)*sin(al_1)*sin(al_2)) - cos(th0_3 +
337 th_3)*sin(al_3)*(sin(th0_1 + th_1)*cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_2)*sin(al_1)) + sin(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(th0_3 + th_3)*sin(al_1)*sin(al_3)) +
d_5*(cos(al_4)*(cos(al_3)*(sin(th0_1 +
338 th_1)*cos(al_1)*cos(al_2) - cos(th0_2 + th_2)*sin(th0_1 +
th_1)*sin(al_1)*sin(al_2)) - cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
th_1)*cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_2)*sin(al_1)) + sin(th0_1 + th_1)*sin(th0_2 +
339 th_2)*sin(th0_3 + th_3)*sin(al_1)*sin(al_3)) + sin(th0_4 +
th_4)*sin(al_4)*(sin(th0_3 + th_3)*(sin(th0_1 +
th_1)*cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_1 +
th_1)*sin(th0_2 +
340 th_2)*sin(al_1)) - cos(th0_4 + th_4)*sin(al_4)*(sin(al_3)*(sin(th0_1 +
th_1)*cos(al_1)*cos(al_2) - cos(th0_2 + th_2)*sin(th0_1 +
th_1)*sin(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +

```









```

+ th_3)*sin(al_3)*(sin(th0_1 + th_1)*cos(al_1)*sin(al_2) + cos(th0_2
+ th_2)*sin(th0_1 + th_1)*cos(al_2)*sin(al_1)) +
371 sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(th0_3 +
th_3)*sin(al_1)*sin(al_3)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
th_3)*(sin(th0_1 + th_1)*cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_2)*sin(al_1)) + cos(th0_3 +
th_3)*sin(th0_1 +
372 th_1)*sin(th0_2 + th_2)*sin(al_1)) + cos(th0_4 +
th_4)*cos(al_4)*(sin(al_3)*(sin(th0_1 + th_1)*cos(al_1)*cos(al_2) -
cos(th0_2 + th_2)*sin(th0_1 + th_1)*sin(al_1)*sin(al_2)) + cos(th0_3
+ th_3)*cos(al_3)*(sin(th0_1 + th_1)*cos(al_1)*sin(al_2) + cos(th0_2
373 + th_2)*sin(th0_1 + th_1)*cos(al_2)*sin(al_1)) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1))) +
a_3*sin(th0_3 + th_3)*(sin(th0_1 + th_1)*cos(al_1)*sin(al_2) +
cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_2)*sin(al_1)) +
374 a_4*sin(th0_4 + th_4)*(sin(al_3)*(sin(th0_1 + th_1)*cos(al_1)*cos(al_2) -
cos(th0_2 + th_2)*sin(th0_1 + th_1)*sin(al_1)*sin(al_2)) + cos(th0_3
+ th_3)*cos(al_3)*(sin(th0_1 + th_1)*cos(al_1)*sin(al_2) + cos(th0_2
+ th_2)*sin(th0_1 +
375 th_1)*cos(al_2)*sin(al_1)) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1)) + a_2*sin(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_1) + a_3*cos(th0_3 + th_3)*sin(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_1) ;
376
377 J_out(1,20)= d_4*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +
378 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2))) + d_3*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) +
379 d_5*(cos(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2)
- sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1
+ th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1
+ th_1)*sin(th0_2 + th_2)*sin(al_2) +
380 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2))) - cos(th0_4 +
th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1

```

381 + th\_1)\*cos(al\_1)\*cos(al\_2)) + cos(th0\_3 + th\_3)\*cos(al\_3)\*(cos(th0\_1 +  
th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) + sin(th0\_1 +  
th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
th\_1)\*cos(al\_1)\*sin(al\_2))) + sin(th0\_3 + th\_3)\*sin(th0\_4 +  
382 th\_4)\*sin(al\_4)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) +  
sin(th0\_1 + th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
th\_1)\*cos(al\_1)\*sin(al\_2))) +  
d\_6\*(cos(al\_5)\*(cos(al\_4)\*(cos(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 +  
th\_2)\*cos(al\_2) -  
383 sin(th0\_1 + th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
th\_1)\*cos(al\_1)\*cos(al\_2)) - cos(th0\_3 + th\_3)\*sin(al\_3)\*(cos(th0\_1 +  
th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) + sin(th0\_1 +  
th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
384 th\_1)\*cos(al\_1)\*sin(al\_2))) - cos(th0\_4 +  
th\_4)\*sin(al\_4)\*(sin(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 +  
th\_2)\*cos(al\_2) - sin(th0\_1 + th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 +  
th\_2)\*sin(th0\_1 + th\_1)\*cos(al\_1)\*cos(al\_2)) + cos(th0\_3 +  
th\_3)\*cos(al\_3)\*(cos(th0\_1  
385 + th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) + sin(th0\_1 +  
th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
th\_1)\*cos(al\_1)\*sin(al\_2))) + sin(th0\_3 + th\_3)\*sin(th0\_4 +  
th\_4)\*sin(al\_4)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) +  
sin(th0\_1 +  
386 th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
th\_1)\*cos(al\_1)\*sin(al\_2))) + sin(th0\_5 + th\_5)\*sin(al\_5)\*(sin(th0\_4  
+ th\_4)\*(sin(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_2) -  
sin(th0\_1 + th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 +  
387 th\_2)\*sin(th0\_1 + th\_1)\*cos(al\_1)\*cos(al\_2)) + cos(th0\_3 +  
th\_3)\*cos(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) +  
sin(th0\_1 + th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
th\_1)\*cos(al\_1)\*sin(al\_2))) + cos(th0\_4 + th\_4)\*sin(th0\_3 +  
388 th\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) + sin(th0\_1 +  
th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
th\_1)\*cos(al\_1)\*sin(al\_2))) - cos(th0\_5 +  
th\_5)\*sin(al\_5)\*(sin(al\_4)\*(cos(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 +  
th\_2)\*cos(al\_2) -  
389 sin(th0\_1 + th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
th\_1)\*cos(al\_1)\*cos(al\_2)) - cos(th0\_3 + th\_3)\*sin(al\_3)\*(cos(th0\_1 +  
th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) + sin(th0\_1 +  
th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
390 th\_1)\*cos(al\_1)\*sin(al\_2))) + cos(th0\_4 +  
th\_4)\*cos(al\_4)\*(sin(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 +



```

th_1)*cos(al_1)*sin(al_2))) - cos(th0_4 +
400 th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2)
- sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1
+ th_1)*cos(al_1)*cos(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1
+ th_1)*sin(th0_2 + th_2)*sin(al_2) +
401 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2))) + sin(th0_3 + th_3)*sin(th0_4 +
th_4)*sin(al_4)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +
sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
402 th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2))) - sin(th0_5 +
th_5)*cos(al_5)*(sin(th0_4 + th_4)*(sin(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) +
403 cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2))) + cos(th0_4 +
th_4)*sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
404 th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2))) + cos(th0_5 +
th_5)*cos(al_5)*(sin(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
405 + cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +
sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2))) + cos(th0_4 +
406 th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2)
- sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1
+ th_1)*cos(al_1)*cos(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1
+ th_1)*sin(th0_2 + th_2)*sin(al_2) +
407 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2))) - sin(th0_3 + th_3)*sin(th0_4 +
th_4)*cos(al_4)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +
sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
408 th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2))) + a_3*sin(th0_3 +
th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + a_6*cos(th0_6 + th_6)*(sin(th0_5 +
409 th_5)*(sin(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +

```

```

410 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*sin(al_2))) + cos(th0_4 +
    th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
    th_2)*sin(th0_1
411 + th_1)*cos(al_1)*cos(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
    th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*sin(al_2))) - sin(th0_3 + th_3)*sin(th0_4 +
412 th_4)*cos(al_4)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +
    sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*sin(al_2))) + cos(th0_5 + th_5)*(sin(th0_4 +
    th_4)*(sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
413 th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
    th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) + cos(th0_3 +
    th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +
    sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
414 th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2))) + cos(th0_4 +
    th_4)*sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
    th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2))) + a_4*sin(th0_4 +
415 th_4)*(sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
    sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*cos(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
416 th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*sin(al_2))) + a_4*cos(th0_4 + th_4)*sin(th0_3 +
    th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
    th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
417 th_1)*cos(al_1)*sin(al_2)) ;
418
419 J_out(1,21)= d_5*(cos(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
    th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
420 th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 +
    th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
    sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*cos(al_2))) - cos(th0_4 +
421 th_4)*sin(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2)
    + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1
    + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1

```

+ th\_1)\*cos(th0\_2 + th\_2) - sin(th0\_1 +  
 422 th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_1)) + cos(th0\_3 +  
 th\_3)\*sin(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_2) -  
 sin(th0\_1 + th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
 th\_1)\*cos(al\_1)\*cos(al\_2))) + d\_4\*(sin(th0\_3 +  
 423 th\_3)\*cos(al\_3)\*(cos(th0\_1 + th\_1)\*cos(th0\_2 + th\_2) - sin(th0\_1 +  
 th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_1)) - sin(al\_3)\*(cos(th0\_1 +  
 th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) + sin(th0\_1 +  
 th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
 424 th\_1)\*cos(al\_1)\*sin(al\_2)) + cos(th0\_3 + th\_3)\*cos(al\_3)\*(cos(th0\_1 +  
 th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_2) - sin(th0\_1 +  
 th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
 th\_1)\*cos(al\_1)\*cos(al\_2))) + d\_6\*(cos(al\_5)\*(cos(al\_4)\*(sin(th0\_3 +  
 425 th\_3)\*cos(al\_3)\*(cos(th0\_1 + th\_1)\*cos(th0\_2 + th\_2) - sin(th0\_1 +  
 th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_1)) - sin(al\_3)\*(cos(th0\_1 +  
 th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) + sin(th0\_1 +  
 th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
 426 th\_1)\*cos(al\_1)\*sin(al\_2)) + cos(th0\_3 + th\_3)\*cos(al\_3)\*(cos(th0\_1 +  
 th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_2) - sin(th0\_1 +  
 th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
 th\_1)\*cos(al\_1)\*cos(al\_2))) - cos(th0\_4 +  
 th\_4)\*sin(al\_4)\*(cos(al\_3)\*(cos(th0\_1  
 427 + th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) + sin(th0\_1 +  
 th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
 th\_1)\*cos(al\_1)\*sin(al\_2)) + sin(th0\_3 + th\_3)\*sin(al\_3)\*(cos(th0\_1 +  
 th\_1)\*cos(th0\_2 + th\_2) - sin(th0\_1 + th\_1)\*sin(th0\_2 +  
 th\_2)\*cos(al\_1)) +  
 428 cos(th0\_3 + th\_3)\*sin(al\_3)\*(cos(th0\_1 + th\_1)\*sin(th0\_2 +  
 th\_2)\*cos(al\_2) - sin(th0\_1 + th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 +  
 th\_2)\*sin(th0\_1 + th\_1)\*cos(al\_1)\*cos(al\_2))) - cos(th0\_5 +  
 th\_5)\*sin(al\_5)\*(sin(al\_4)\*(sin(th0\_3 +  
 429 th\_3)\*cos(al\_3)\*(cos(th0\_1 + th\_1)\*cos(th0\_2 + th\_2) - sin(th0\_1 +  
 th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_1)) - sin(al\_3)\*(cos(th0\_1 +  
 th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) + sin(th0\_1 +  
 th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
 430 th\_1)\*cos(al\_1)\*sin(al\_2)) + cos(th0\_3 + th\_3)\*cos(al\_3)\*(cos(th0\_1 +  
 th\_1)\*sin(th0\_2 + th\_2)\*cos(al\_2) - sin(th0\_1 +  
 th\_1)\*sin(al\_1)\*sin(al\_2) + cos(th0\_2 + th\_2)\*sin(th0\_1 +  
 th\_1)\*cos(al\_1)\*cos(al\_2))) + cos(th0\_4 +  
 th\_4)\*cos(al\_4)\*(cos(al\_3)\*(cos(th0\_1  
 431 + th\_1)\*sin(th0\_2 + th\_2)\*sin(al\_2) + sin(th0\_1 +  
 th\_1)\*cos(al\_2)\*sin(al\_1) + cos(th0\_2 + th\_2)\*sin(th0\_1 +



```

441 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))
    - sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +
    sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 +
442 th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
    sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*cos(al_2))) - cos(th0_4 +
    th_4)*sin(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*sin(al_2) +
443 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
    th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)) + cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
444 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
    + cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)))) +
    cos(th0_5 + th_5)*cos(al_5)*(sin(al_4)*(sin(th0_3 +
    th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
445 th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
    th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
    th_1)*sin(th0_2 +
446 th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
    th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) + cos(th0_4 +
    th_4)*cos(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2
447 + th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
    th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
    th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
    sin(th0_1 +
448 th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*cos(al_2)))) - sin(th0_4 + th_4)*sin(th0_5 +
    th_5)*cos(al_5)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
449 th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
    th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
    th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
    sin(th0_1 +
450 th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*cos(al_2)))) + a_4*sin(th0_4 +

```



```

th_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +
sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
451 th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) + cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
452 th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) + a_5*sin(th0_5 +
th_5)*(sin(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) -
453 sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
454 th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) + cos(th0_4 +
th_4)*cos(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*sin(th0_1 +
455 th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) + cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
456 th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) + a_5*cos(th0_5 + th_5)*sin(th0_4 +
th_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +
sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
457 th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 +
458 th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) ;
459
460 J_out(1,22)= a_5*sin(th0_5 + th_5)*(cos(al_4)*(cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
461 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))
+ cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +

```





```

th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1)
481 + cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
482 sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) - sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3
+ th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
483 th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
th_4)*sin(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) -
484 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
485 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
+ cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)))) -
d_5*(sin(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) +
486 cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
487 sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3
+ th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
488 th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1))) - cos(th0_4 +
th_4)*cos(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) -
489 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
490 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
+ cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)))) +
a_6*cos(th0_6 + th_6)*sin(th0_5 +

```

```

th_5)*(cos(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) +
491 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) + cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
492 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
+ cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) -
sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
493 th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1))) + cos(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
494 th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
495 th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)))) ;
496
497 J_out(1,23)= - d_6*(sin(al_5)*(cos(al_4)*(cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
498 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))
+ cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) -
499 sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) -
500 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
th_4)*sin(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) +

```





```

th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) +
522 cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) + cos(th0_4 +
523 th_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 +
524 th_1)*sin(th0_2 + th_2)*cos(al_1)))));
525
526 J_out(1,24)= 0 ;
527
528 J_out(2,1)= d_5*(cos(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) -
529 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) - sin(th0_4 +
530 th_4)*sin(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
531 th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
th_4)*sin(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 +
532 th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
533 th_1)*cos(al_1)*cos(al_2)))) - d_6*(cos(th0_5 +
th_5)*sin(al_5)*(sin(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
534 th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -

```







```

th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
556 th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2))) + cos(th0_4 + th_4)*(sin(th0_3 +
th_3)*(cos(th0_1 +
557 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
+ cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) -
cos(th0_3 + th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)))) +
558 a_6*cos(th0_6 + th_6)*(sin(th0_5 + th_5)*(sin(al_4)*(cos(al_3)*(cos(th0_1
+ th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
559 th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))
+ cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) +
560 sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) -
561 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))) - cos(th0_4 +
th_4)*cos(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) +
562 sin(th0_1 + th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
563 th_1)*cos(al_1)*cos(al_2)))) - cos(th0_5 + th_5)*(sin(th0_4 +
th_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 +
th_2) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) -
sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
564 th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +

```



```

575 th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
    th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) + cos(th0_5 +
    th_5)*cos(al_5)*(sin(al_4)*(cos(al_3)*(cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*sin(al_2) + sin(th0_1 + th_1)*cos(al_2)*sin(al_1)
576 + cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*sin(al_2)) + sin(th0_3 +
    th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
    th_3)*sin(al_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) -
577 sin(th0_1 + th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3
    + th_3)*(cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 +
    th_1)*sin(al_1)*sin(al_2) + cos(th0_2 +
578 th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 +
    th_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) - sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1))) - cos(th0_4 +
    th_4)*cos(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
    th_1)*cos(th0_2 + th_2) -
579 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
    th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
580 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
    + cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) +
    sin(th0_5 + th_5)*cos(al_5)*(sin(th0_4 + th_4)*(sin(th0_3 +
    th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(th0_2 + th_2) -
581 sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(al_3)*(cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*sin(al_2) + sin(th0_1 +
    th_1)*cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
582 th_1)*sin(th0_2 + th_2)*cos(al_2) - sin(th0_1 + th_1)*sin(al_1)*sin(al_2)
    + cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_1)*cos(al_2))) +
    cos(th0_4 + th_4)*(sin(th0_3 + th_3)*(cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_2) - sin(th0_1 +
583 th_1)*sin(al_1)*sin(al_2) + cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_1 +
    th_1)*cos(th0_2 + th_2) - sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)))) - a_2*sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1) ;
584
585 J_out(2,2)= d_3*(cos(th0_2 + th_2)*sin(th0_1 + th_1)*sin(al_2) +
    cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)*sin(al_2)) +
    d_4*(cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1)*sin(al_2) +

```





```

606 th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_2) +
    cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)*cos(al_2)) + sin(th0_3
    + th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2) - cos(th0_1 +
    th_1)*cos(th0_2 + th_2)*cos(al_1))) -
607 sin(al_5)*(sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 + th_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1)*cos(al_2) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) + cos(th0_3 + th_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2) - cos(th0_1 + th_1)*cos(th0_2 +
608 th_2)*cos(al_1))) - cos(al_4)*(cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*sin(al_2) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1)*cos(al_2) + cos(th0_1 + th_1)*sin(th0_2 +
609 th_2)*cos(al_1)*cos(al_2)) - sin(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1))) + cos(th0_4 + th_4)*sin(al_4)*(sin(al_3)*(cos(th0_2
    + th_2)*sin(th0_1 + th_1)*sin(al_2) + cos(th0_1 +
610 th_1)*sin(th0_2 + th_2)*cos(al_1)*sin(al_2)) - cos(th0_3 +
    th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_2) +
    cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)*cos(al_2)) + sin(th0_3
    + th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2) -
611 cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1))) + cos(th0_5 +
    th_5)*cos(al_5)*(sin(al_4)*(cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*sin(al_2) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(cos(th0_2 +
612 th_2)*sin(th0_1 + th_1)*cos(al_2) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) - sin(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1))) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
613 th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_2) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)*cos(al_2)) + cos(th0_3 +
    th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2) - cos(th0_1 +
    th_1)*cos(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
614 th_4)*cos(al_4)*(sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1)*sin(al_2)
    + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)*sin(al_2)) -
    cos(th0_3 + th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_2) + cos(th0_1 + th_1)*sin(th0_2 +
615 th_2)*cos(al_1)*cos(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1))) - a_3*cos(th0_3 + th_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1))

```





```

626 th_4)*cos(al_4)*(sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1)*sin(al_2)
    + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)*sin(al_2)) -
    cos(th0_3 + th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_2) + cos(th0_1 + th_1)*sin(th0_2 +
627 th_2)*cos(al_1)*cos(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1))) - a_3*sin(th0_3 + th_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1)*cos(al_2) + cos(th0_1 + th_1)*sin(th0_2 +
628 th_2)*cos(al_1)*cos(al_2)) - a_4*cos(th0_4 + th_4)*(sin(th0_3 +
    th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1)*cos(al_2) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)*cos(al_2)) + cos(th0_3 +
    th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2) - cos(th0_1 +
629 th_1)*cos(th0_2 + th_2)*cos(al_1))) - a_5*cos(th0_5 + th_5)*(cos(th0_4 +
    th_4)*(sin(th0_3 + th_3)*(cos(th0_2 + th_2)*sin(th0_1 +
    th_1)*cos(al_2) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) + cos(th0_3 + th_3)*(sin(th0_1 +
    th_1)*sin(th0_2 +
630 th_2) - cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1))) - sin(th0_4 +
    th_4)*(sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1)*sin(al_2) +
    cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)*sin(al_2)) - cos(th0_3
    + th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 +
631 th_1)*cos(al_2) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1))) + a_2*cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1) ;
632
633 J_out(2,3)= d_4*(cos(th0_3 + th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1
    + th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3
    + th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
    cos(th0_1 + th_1)*sin(al_1)*sin(al_2) -
634 cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) -
    d_6*(sin(th0_5 + th_5)*sin(al_5)*(cos(th0_4 + th_4)*(sin(th0_3 +
    th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 + th_3)*(sin(th0_1 +
635 th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2)
    - cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) +
    sin(th0_4 + th_4)*(cos(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
636 th_2)*cos(al_1)) - sin(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +

```





```

657 th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) - cos(th0_4 +
    th_4)*cos(al_4)*(cos(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)) - sin(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 +
658 th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
    th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)))) - a_4*sin(th0_4 +
    th_4)*(cos(th0_3 + th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 +
    th_1) + cos(th0_1 + th_1)*sin(th0_2 +
659 th_2)*cos(al_1)) - sin(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) - a_3*sin(th0_3 + th_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1)
660 + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) + a_6*cos(th0_6 +
    th_6)*(sin(th0_5 + th_5)*(sin(al_4)*(cos(th0_3 +
    th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3 +
661 th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
    cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3
    + th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) +
662 cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
    th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) - cos(th0_4 + th_4)*cos(al_4)*(cos(th0_3
663 + th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3 +
    th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
    cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
664 th_2)*cos(al_1)*cos(al_2)))) - cos(th0_5 + th_5)*(cos(th0_4 +
    th_4)*(sin(th0_3 + th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) +
    cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
    th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
665 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*(cos(th0_3 +
    th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3 +
666 th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
    cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)))) - a_3*cos(th0_3 + th_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +

```

```

667 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) + a_5*sin(th0_5 +
    th_5)*(sin(al_4)*(cos(th0_3 + th_3)*sin(al_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)) - sin(th0_3 +
668 th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
    cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3
    + th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) +
669 cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
    th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) - cos(th0_4 + th_4)*cos(al_4)*(cos(th0_3
670 + th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3 +
    th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
    cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
671 th_2)*cos(al_1)*cos(al_2)))) - a_4*cos(th0_4 + th_4)*(sin(th0_3 +
    th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 + th_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
672 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) - a_5*cos(th0_5 + th_5)*(cos(th0_4 +
    th_4)*(sin(th0_3 + th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) +
    cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
673 th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*(cos(th0_3 +
    th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
674 th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3 +
    th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
    cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)))) ;
675
676 J_out(2,4)= a_5*cos(th0_5 + th_5)*(sin(th0_4 + th_4)*(sin(th0_3 +
    th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
677 th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)))
    - cos(th0_4 + th_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1)
    - sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
    th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) +

```



```

688 th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
689 th_2)*cos(al_1)))) - d_5*(cos(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
    th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
690 th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)))
    + sin(th0_4 + th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1 +
    th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
691 th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)) + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
692 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)))) - a_4*cos(th0_4 +
    th_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) - sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
693 th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)) + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
694 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) + a_5*sin(th0_5 + th_5)*(sin(th0_4 +
    th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
    sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
695 th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +
    th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
    th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
    cos(th0_1 +
696 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) + cos(th0_4 + th_4)*cos(al_4)*(sin(th0_3
    + th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
697 th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)))) + a_6*cos(th0_6 + th_6)*(sin(th0_5 +
    th_5)*(sin(th0_4 + th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1 +

```



```

698 th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2)
    + cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) +
    sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) +
    cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) +
699 cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
    th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) + cos(th0_4 +
    th_4)*cos(al_4)*(sin(th0_3 + th_3)*(sin(th0_1 + th_1)*sin(th0_2 +
700 th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
    th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 +
    th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_5 + th_5)*(sin(th0_4 +
701 th_4)*(sin(th0_3 + th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
    cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 +
702 th_1)*sin(th0_2 + th_2)*cos(al_1))) - cos(th0_4 +
    th_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) - sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
703 th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))
    + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
    th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)))) +
704 a_4*sin(th0_4 + th_4)*(sin(th0_3 + th_3)*(sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
    th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 +
    th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1
705 + th_1)*sin(th0_2 + th_2)*cos(al_1))) - a_6*sin(th0_6 +
    th_6)*(sin(al_5)*(cos(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
    th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
706 th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1))) + sin(th0_4 + th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1
    + th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
707 th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)) + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 +

```

```

708 th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
    th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) + sin(th0_5 +
    th_5)*cos(al_5)*(sin(th0_4 + th_4)*(sin(th0_3 + th_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
709 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1))) - cos(th0_4 + th_4)*(sin(al_3)*(cos(th0_1 +
710 th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2)
    + cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) +
    sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) +
    cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) +
711 cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
    th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) - cos(th0_5 +
    th_5)*cos(al_5)*(sin(th0_4 +
712 th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
    sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
    th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +
    th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
713 th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
    th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
    cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) + cos(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
714 th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
715 th_2)*cos(al_1)))));
716
717 J_out(2,5)= d_6*(sin(th0_5 + th_5)*sin(al_5)*(sin(al_4)*(sin(th0_3 +
    th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
    th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
718 th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) +
719 sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 + th_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +

```







```

750 th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
      th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_1)))) ;
751
752 J_out(2,6)= a_6*cos(th0_6 + th_6)*(sin(al_5)*(cos(al_4)*(sin(th0_3 +
      th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
753 th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
      th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2))) -
754 sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 + th_3)*(sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
      th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
      th_2)*sin(th0_1 + th_1) +
755 cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
      sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
      th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +
756 th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
      th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
      cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
757 th_2)*cos(al_1)*cos(al_2)))) + cos(th0_5 +
      th_5)*cos(al_5)*(sin(al_4)*(sin(th0_3 + th_3)*sin(al_3)*(cos(th0_2 +
      th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
      sin(th0_1 +
758 th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
      th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
759 th_2)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
      th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
      th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
760 th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)))
      - cos(th0_4 + th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1 +
      th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +

```

```

th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
761 th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
762 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) + sin(th0_5 + th_5)*cos(al_5)*(sin(th0_4
+ th_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) - sin(th0_1
+ th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
763 th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +
th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 +
764 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) + cos(th0_4 + th_4)*(sin(th0_3 +
th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
765 th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)))) - a_6*sin(th0_6 + th_6)*(sin(th0_5 +
th_5)*(sin(al_4)*(sin(th0_3 + th_3)*sin(al_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1)
766 + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
767 th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2)
- cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) +
sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 + th_3)*(sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
768 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1))) - cos(th0_4 + th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1
+
769 th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2)
+ cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) +
sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) +
cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) +
770 cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +

```

```

th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) - cos(th0_5 +
th_5)*(sin(th0_4 + th_4)*(sin(al_3)*(cos(th0_1 +
771 th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2)
+ cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) +
sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) +
cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) +
772 cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) + cos(th0_4 +
th_4)*(sin(th0_3 + th_3)*(sin(th0_1 + th_1)*sin(th0_2 +
773 th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)))));
774
775 J_out(2,7)= 0 ;
776
777 J_out(2,8)= -cos(th0_1 + th_1)*sin(al_1) ;
778
779 J_out(2,9)= sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) - cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2) ;
780
781 J_out(2,10)= sin(th0_3 + th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 +
th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) -
cos(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2
782 + th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) ;
783
784 J_out(2,11)= cos(al_4)*(sin(th0_3 + th_3)*sin(al_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
785 th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) - sin(th0_4 +
786 th_4)*sin(al_4)*(sin(th0_3 + th_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +

```



```

th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
787 th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +
788 th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
789 th_2)*cos(al_1)*cos(al_2))) ;
790
791 J_out(2,12)= cos(al_5)*(cos(al_4)*(sin(th0_3 + th_3)*sin(al_3)*(cos(th0_2
+ th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) +
792 cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) - sin(th0_4 +
793 th_4)*sin(al_4)*(sin(th0_3 + th_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
794 th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +
795 th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
796 th_2)*cos(al_1)*cos(al_2))) - cos(th0_5 +
th_5)*sin(al_5)*(sin(al_4)*(sin(th0_3 + th_3)*sin(al_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
sin(th0_1 +
797 th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +

```

```

798 th_2)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
    th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
799 th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)))
    - cos(th0_4 + th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1 +
    th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
    th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
800 th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)) + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
801 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2)))) - sin(th0_5 + th_5)*sin(al_5)*(sin(th0_4
    + th_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) - sin(th0_1
    + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
802 th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +
    th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
    th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
    cos(th0_1 +
803 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
    th_2)*cos(al_1)*cos(al_2))) + cos(th0_4 + th_4)*(sin(th0_3 +
    th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
804 th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
    th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
    th_2)*cos(al_1)))) ;

805
806 J_out(2,13)= sin(th0_1 + th_1) ;
807
808 J_out(2,14)= cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
    th_1)*sin(th0_2 + th_2)*cos(al_1) ;

809
810 J_out(2,15)= cos(th0_3 + th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) +
    cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) - sin(th0_3 +
    th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
    th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
811 th_2)*cos(al_1)*cos(al_2)) ;

812
813 J_out(2,16)= - sin(th0_4 + th_4)*(sin(al_3)*(cos(th0_1 +
    th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +

```

```

th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
th_2)*sin(th0_1 +
814 th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) - cos(th0_4 +
815 th_4)*(sin(th0_3 + th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 +
816 th_1)*sin(th0_2 + th_2)*cos(al_1))) ;
817
818 J_out(2,17)= sin(th0_5 + th_5)*(sin(al_4)*(sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2)
819 + cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) + sin(th0_4 +
820 th_4)*cos(al_4)*(sin(th0_3 + th_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
821 th_1)*sin(th0_2 + th_2)*cos(al_1))) - cos(th0_4 +
th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +
822 th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
823 th_2)*cos(al_1)*cos(al_2)))) - cos(th0_5 + th_5)*(sin(th0_4 +
th_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +
824 th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +

```

```

825 th_2)*cos(al_1)*cos(al_2))) + cos(th0_4 + th_4)*(sin(th0_3 +
      th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
      th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
826 th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_1)))) ;
827
828 J_out(2,18)= cos(th0_6 + th_6)*(sin(th0_5 + th_5)*(sin(al_4)*(sin(th0_3 +
      th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
      th_1)*cos(al_2)*sin(al_1) - sin(th0_1 +
829 th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
      th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
830 th_2)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
      th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
      th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
831 th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)))
      - cos(th0_4 + th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1 +
      th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
      th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
832 th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
      th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_1)) + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
833 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)))) - cos(th0_5 + th_5)*(sin(th0_4 +
      th_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) - sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
834 th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
      th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_1)) + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
835 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2))) + cos(th0_4 + th_4)*(sin(th0_3 +
      th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
      th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
836 th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
      th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_1)))) + sin(th0_6 +

```



```

846 th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2)
      - cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) +
      sin(th0_5 + th_5)*cos(al_5)*(sin(th0_4 + th_4)*(sin(al_3)*(cos(th0_1
      + th_1)*cos(al_2)*sin(al_1) - sin(th0_1 +
847 th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
      th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_1)) + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
848 th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2)
      - cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) +
      cos(th0_4 + th_4)*(sin(th0_3 + th_3)*(sin(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_2) + cos(th0_1 +
849 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
      th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_1)))));
850
851 J_out(2,19)= - d_3*(cos(th0_1 + th_1)*cos(al_1)*cos(al_2) - cos(th0_1 +
      th_1)*cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) -
      d_6*(cos(al_5)*(cos(al_4)*(cos(al_3)*(cos(th0_1 +
      th_1)*cos(al_1)*cos(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) -
852 cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(al_1)*sin(al_2) +
      cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_1
      + th_1)*sin(th0_2 + th_2)*sin(th0_3 + th_3)*sin(al_1)*sin(al_3)) +
      sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
853 th_3)*(cos(th0_1 + th_1)*cos(al_1)*sin(al_2) + cos(th0_1 +
      th_1)*cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_1 +
      th_1)*cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) - cos(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_1)*cos(al_2) -
854 cos(th0_1 + th_1)*cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(al_1)*sin(al_2) + cos(th0_1 +
      th_1)*cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) - cos(th0_1 +
      th_1)*sin(th0_2 + th_2)*sin(th0_3 +
855 th_3)*cos(al_3)*sin(al_1))) - cos(th0_5 +
      th_5)*sin(al_5)*(sin(al_4)*(cos(al_3)*(cos(th0_1 +
      th_1)*cos(al_1)*cos(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
      th_1)*cos(al_1)*sin(al_2) +
856 cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_1 +
      th_1)*sin(th0_2 + th_2)*sin(th0_3 + th_3)*sin(al_1)*sin(al_3)) -
      sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 + th_3)*(cos(th0_1 +

```









```

887 + th_1)*cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) - cos(th0_1 +
      th_1)*sin(th0_2 + th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1))) -
      a_4*cos(th0_4 + th_4)*(sin(th0_3 + th_3)*(cos(th0_1 +
      th_1)*cos(al_1)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
888 th_2)*cos(al_2)*sin(al_1)) + cos(th0_1 + th_1)*cos(th0_3 +
      th_3)*sin(th0_2 + th_2)*sin(al_1)) - a_4*sin(th0_4 +
      th_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_1)*cos(al_2) - cos(th0_1 +
      th_1)*cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
889 th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(al_1)*sin(al_2) + cos(th0_1 +
      th_1)*cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) - cos(th0_1 +
      th_1)*sin(th0_2 + th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1)) -
      a_3*sin(th0_3 + th_3)*(cos(th0_1 + th_1)*cos(al_1)*sin(al_2) +
890 cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) - a_2*cos(th0_1
      + th_1)*sin(th0_2 + th_2)*sin(al_1) - a_3*cos(th0_1 + th_1)*cos(th0_3
      + th_3)*sin(th0_2 + th_2)*sin(al_1) ;

891
892 J_out(2,20)= d_4*(cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
      th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) + cos(th0_3 +
      th_3)*sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) - sin(th0_1
893 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*sin(al_2))) - d_6*(cos(al_5)*(cos(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
      th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
894 th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 +
      th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) - sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*sin(al_2))) - cos(al_4)*(cos(al_3)*(sin(th0_1
895 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
      th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
      th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
      th_2)*sin(al_2) +
896 cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2))) + sin(th0_3 +
      th_3)*sin(th0_4 + th_4)*sin(al_4)*(cos(th0_1 +
      th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
      th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*sin(al_2))) -
897 sin(th0_5 + th_5)*sin(al_5)*(sin(th0_4 + th_4)*(sin(al_3)*(sin(th0_1 +
      th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
      th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +

```







```

th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 +
927 th_2)*cos(al_1)*cos(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2))) + cos(th0_4 +
th_4)*cos(al_4)*(sin(al_3)*(sin(th0_1
928 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*cos(al_3)*(cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) +
929 cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2))) + sin(th0_3 +
th_3)*sin(th0_4 + th_4)*cos(al_4)*(cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2))) +
930 a_4*sin(th0_4 + th_4)*(sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*cos(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
931 sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2))) - a_4*cos(th0_4 +
th_4)*sin(th0_3 + th_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
932 th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) ;
933
934 J_out(2,21)= d_5*(cos(al_4)*(sin(al_3)*(cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) +
935 cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) - cos(th0_4 +
936 th_4)*sin(al_4)*(sin(th0_3 + th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1
+ th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) -
cos(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
937 th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +

```









```

+ th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
968 th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2)
+ cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) +
cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) -
969 cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) - sin(th0_4 +
th_4)*sin(th0_5 + th_5)*cos(al_5)*(sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
970 th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2)
+ cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) +
cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) -
971 cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) +
a_5*cos(th0_5 + th_5)*sin(th0_4 + th_4)*(sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
972 th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2)
+ cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) +
cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) -
973 cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) ;
974
975 J_out(2,22)= a_5*sin(th0_5 + th_5)*(cos(al_4)*(sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
976 th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) -
977 sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 + th_3)*(sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) +
978 cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +

```



```

th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
989 th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
990 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)))) - a_6*sin(th0_6 +
th_6)*(sin(al_5)*(sin(al_4)*(sin(th0_3 + th_3)*sin(al_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) -
991 cos(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) - sin(th0_1 +
th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
992 th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3
+ th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
993 th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1))) - cos(th0_4 + th_4)*cos(al_4)*(sin(al_3)*(cos(th0_1
+ th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
994 th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) + cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 +
th_1)*sin(th0_2 +
995 th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)))) - cos(th0_5 +
th_5)*cos(al_5)*(cos(al_4)*(sin(th0_3 + th_3)*sin(al_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
996 th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
997 cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) - sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3
+ th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
998 th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +

```



```

th_2)*cos(al_1))) + cos(th0_4 + th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1
+
1009 th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2)
+ cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) +
sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) +
cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1)) +
1010 cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)))) ;
1011
1012 J_out(2,23)= - d_6*(sin(al_5)*(cos(al_4)*(sin(th0_3 +
th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2)
1013 + cos(th0_1 + th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*sin(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) - sin(th0_4 +
1014 th_4)*sin(al_4)*(sin(th0_3 + th_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
1015 th_1)*sin(th0_2 + th_2)*cos(al_1))) + cos(th0_4 +
th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
sin(th0_1 + th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 +
1016 th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
1017 th_2)*cos(al_1)*cos(al_2)))) + cos(th0_5 +
th_5)*cos(al_5)*(sin(al_4)*(sin(th0_3 + th_3)*sin(al_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 + th_1)*cos(al_2)*sin(al_1) -
sin(th0_1 +
1018 th_1)*sin(th0_2 + th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
1019 th_2)*cos(al_1)*cos(al_2))) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +

```



```

th_3)*cos(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
1030 th_1)*sin(th0_2 + th_2)*cos(al_1)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) +
cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)))) - cos(al_5)*(cos(al_4)*(sin(th0_3 +
1031 th_3)*sin(al_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_1)) - cos(al_3)*(cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
1032 th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 + th_3)*sin(al_3)*(sin(th0_1 +
th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2))) - sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
1033 th_3)*(sin(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_2) + cos(th0_1 +
th_1)*sin(al_1)*sin(al_2) - cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*(cos(th0_2 +
th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 +
1034 th_2)*cos(al_1))) + cos(th0_4 + th_4)*sin(al_4)*(sin(al_3)*(cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
1035 th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))
+ cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)))) +
1036 sin(th0_5 + th_5)*sin(al_5)*(sin(th0_4 + th_4)*(sin(al_3)*(cos(th0_1 +
th_1)*cos(al_2)*sin(al_1) - sin(th0_1 + th_1)*sin(th0_2 +
th_2)*sin(al_2) + cos(th0_1 + th_1)*cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + sin(th0_3 + th_3)*cos(al_3)*(cos(th0_2 +
1037 th_2)*sin(th0_1 + th_1) + cos(th0_1 + th_1)*sin(th0_2 + th_2)*cos(al_1))
+ cos(th0_3 + th_3)*cos(al_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2))) +
1038 cos(th0_4 + th_4)*(sin(th0_3 + th_3)*(sin(th0_1 + th_1)*sin(th0_2 +
th_2)*cos(al_2) + cos(th0_1 + th_1)*sin(al_1)*sin(al_2) - cos(th0_1 +
th_1)*cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 +
th_3)*(cos(th0_2 + th_2)*sin(th0_1 + th_1) + cos(th0_1 +
1039 th_1)*sin(th0_2 + th_2)*cos(al_1)))));
1040
1041 J_out(2,24)= 0 ;
1042
1043 J_out(3,1)= 0 ;

```



```

1044
1045 J_out(3,2)= d_4*(cos(th0_2 + th_2)*sin(th0_3 + th_3)*sin(al_1)*sin(al_3)
      + sin(th0_2 + th_2)*cos(al_3)*sin(al_1)*sin(al_2) + cos(th0_3 +
      th_3)*sin(th0_2 + th_2)*cos(al_2)*sin(al_1)*sin(al_3)) +
      d_5*(cos(al_4)*(cos(th0_2 + th_2)*sin(th0_3 +
1046 th_3)*sin(al_1)*sin(al_3) + sin(th0_2 +
      th_2)*cos(al_3)*sin(al_1)*sin(al_2) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_2)*sin(al_1)*sin(al_3)) + sin(th0_4 +
      th_4)*sin(al_4)*(cos(th0_2 + th_2)*cos(th0_3 + th_3)*sin(al_1) -
      sin(th0_2 + th_2)*sin(th0_3 +
1047 th_3)*cos(al_2)*sin(al_1)) + cos(th0_4 + th_4)*sin(al_4)*(cos(th0_2 +
      th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1) - sin(th0_2 +
      th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_2)*cos(al_3)*sin(al_1))) +
1048 d_6*(cos(al_5)*(cos(al_4)*(cos(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3) + sin(th0_2 +
      th_2)*cos(al_3)*sin(al_1)*sin(al_2) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_2)*sin(al_1)*sin(al_3)) + sin(th0_4 +
      th_4)*sin(al_4)*(cos(th0_2 +
1049 th_2)*cos(th0_3 + th_3)*sin(al_1) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_2)*sin(al_1)) + cos(th0_4 + th_4)*sin(al_4)*(cos(th0_2 +
      th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1) - sin(th0_2 +
      th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 +
1050 th_3)*sin(th0_2 + th_2)*cos(al_2)*cos(al_3)*sin(al_1))) + sin(th0_5 +
      th_5)*sin(al_5)*(cos(th0_4 + th_4)*(cos(th0_2 + th_2)*cos(th0_3 +
      th_3)*sin(al_1) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_2)*sin(al_1)) - sin(th0_4 + th_4)*(cos(th0_2 +
1051 th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1) - sin(th0_2 +
      th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_2)*cos(al_3)*sin(al_1))) + cos(th0_5 +
      th_5)*sin(al_5)*(sin(th0_4 + th_4)*cos(al_4)*(cos(th0_2 +
      th_2)*cos(th0_3 +
1052 th_3)*sin(al_1) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_2)*sin(al_1)) - sin(al_4)*(cos(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3) + sin(th0_2 +
      th_2)*cos(al_3)*sin(al_1)*sin(al_2) + cos(th0_3 + th_3)*sin(th0_2 +
1053 th_2)*cos(al_2)*sin(al_1)*sin(al_3)) + cos(th0_4 +
      th_4)*cos(al_4)*(cos(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1) - sin(th0_2 +
      th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_2)*cos(al_3)*sin(al_1)))) -

```

```

1054 a_5*sin(th0_5 + th_5)*(sin(th0_4 + th_4)*cos(al_4)*(cos(th0_2 +
      th_2)*cos(th0_3 + th_3)*sin(al_1) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_2)*sin(al_1)) - sin(al_4)*(cos(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3) + sin(th0_2 +
1055 th_2)*cos(al_3)*sin(al_1)*sin(al_2) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_2)*sin(al_1)*sin(al_3)) + cos(th0_4 +
      th_4)*cos(al_4)*(cos(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1) - sin(th0_2 +
      th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 +
1056 th_3)*sin(th0_2 + th_2)*cos(al_2)*cos(al_3)*sin(al_1))) + a_2*cos(th0_2 +
      th_2)*sin(al_1) - a_6*sin(th0_6 + th_6)*(sin(th0_5 +
      th_5)*cos(al_5)*(cos(th0_4 + th_4)*(cos(th0_2 + th_2)*cos(th0_3 +
      th_3)*sin(al_1) - sin(th0_2 + th_2)*sin(th0_3 +
1057 th_3)*cos(al_2)*sin(al_1)) - sin(th0_4 + th_4)*(cos(th0_2 +
      th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1) - sin(th0_2 +
      th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_2)*cos(al_3)*sin(al_1))) -
1058 sin(al_5)*(cos(al_4)*(cos(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3) + sin(th0_2 +
      th_2)*cos(al_3)*sin(al_1)*sin(al_2) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_2)*sin(al_1)*sin(al_3)) + sin(th0_4 +
      th_4)*sin(al_4)*(cos(th0_2 + th_2)*cos(th0_3
1059 + th_3)*sin(al_1) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_2)*sin(al_1)) + cos(th0_4 + th_4)*sin(al_4)*(cos(th0_2 +
      th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1) - sin(th0_2 +
      th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 + th_3)*sin(th0_2 +
1060 th_2)*cos(al_2)*cos(al_3)*sin(al_1))) + cos(th0_5 +
      th_5)*cos(al_5)*(sin(th0_4 + th_4)*cos(al_4)*(cos(th0_2 +
      th_2)*cos(th0_3 + th_3)*sin(al_1) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_2)*sin(al_1)) - sin(al_4)*(cos(th0_2 + th_2)*sin(th0_3 +
1061 th_3)*sin(al_1)*sin(al_3) + sin(th0_2 +
      th_2)*cos(al_3)*sin(al_1)*sin(al_2) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_2)*sin(al_1)*sin(al_3)) + cos(th0_4 +
      th_4)*cos(al_4)*(cos(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1) - sin(th0_2 +
1062 th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_2)*cos(al_3)*sin(al_1))) + a_4*cos(th0_4 +
      th_4)*(cos(th0_2 + th_2)*cos(th0_3 + th_3)*sin(al_1) - sin(th0_2 +
      th_2)*sin(th0_3 + th_3)*cos(al_2)*sin(al_1)) + a_5*cos(th0_5 +
1063 th_5)*(cos(th0_4 + th_4)*(cos(th0_2 + th_2)*cos(th0_3 + th_3)*sin(al_1) -
      sin(th0_2 + th_2)*sin(th0_3 + th_3)*cos(al_2)*sin(al_1)) - sin(th0_4
      + th_4)*(cos(th0_2 + th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1) -

```

```

sin(th0_2 +
1064 th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 + th_3)*sin(th0_2 +
th_2)*cos(al_2)*cos(al_3)*sin(al_1))) - a_4*sin(th0_4 +
th_4)*(cos(th0_2 + th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1) -
sin(th0_2 + th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 +
1065 th_3)*sin(th0_2 + th_2)*cos(al_2)*cos(al_3)*sin(al_1)) + a_6*cos(th0_6 +
th_6)*(cos(th0_5 + th_5)*(cos(th0_4 + th_4)*(cos(th0_2 +
th_2)*cos(th0_3 + th_3)*sin(al_1) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_2)*sin(al_1)) - sin(th0_4 + th_4)*(cos(th0_2 +
1066 th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1) - sin(th0_2 +
th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 + th_3)*sin(th0_2 +
th_2)*cos(al_2)*cos(al_3)*sin(al_1))) - sin(th0_5 + th_5)*(sin(th0_4
+ th_4)*cos(al_4)*(cos(th0_2 + th_2)*cos(th0_3 +
1067 th_3)*sin(al_1) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_2)*sin(al_1)) - sin(al_4)*(cos(th0_2 + th_2)*sin(th0_3 +
th_3)*sin(al_1)*sin(al_3) + sin(th0_2 +
th_2)*cos(al_3)*sin(al_1)*sin(al_2) + cos(th0_3 + th_3)*sin(th0_2 +
1068 th_2)*cos(al_2)*sin(al_1)*sin(al_3)) + cos(th0_4 +
th_4)*cos(al_4)*(cos(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1) - sin(th0_2 +
th_2)*sin(al_1)*sin(al_2)*sin(al_3) + cos(th0_3 + th_3)*sin(th0_2 +
th_2)*cos(al_2)*cos(al_3)*sin(al_1)))) +
1069 d_3*sin(th0_2 + th_2)*sin(al_1)*sin(al_2) + a_3*cos(th0_2 +
th_2)*cos(th0_3 + th_3)*sin(al_1) - a_3*sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_2)*sin(al_1) ;
1070
1071 J_out(3,3)= d_4*(sin(th0_3 + th_3)*sin(al_3)*(cos(al_1)*sin(al_2) +
cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2
+ th_2)*sin(al_1)*sin(al_3)) + d_5*(cos(al_4)*(sin(th0_3 +
th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
1072 th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
th_2)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*sin(al_4)*(cos(th0_3 +
th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) -
sin(th0_2 + th_2)*sin(th0_3 + th_3)*sin(al_1)) +
1073 cos(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
th_2)*cos(al_3)*sin(al_1))) + d_6*(cos(al_5)*(cos(al_4)*(sin(th0_3 +
1074 th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
th_2)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*sin(al_4)*(cos(th0_3 +
th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) -

```



```

1085 sin(th0_2 + th_2)*sin(th0_3 + th_3)*sin(al_1)) + cos(th0_4 +
      th_4)*sin(al_4)*(sin(th0_3 + th_3)*cos(al_3)*(cos(al_1)*sin(al_2) +
      cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2
      + th_2)*cos(al_3)*sin(al_1))) + cos(th0_5 +
1086 th_5)*cos(al_5)*(cos(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_3)*sin(al_1)) - sin(al_4)*(sin(th0_3 +
1087 th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*cos(al_4)*(cos(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) -
1088 sin(th0_2 + th_2)*sin(th0_3 + th_3)*sin(al_1))) + a_3*cos(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      a_4*cos(th0_4 + th_4)*(cos(th0_3 + th_3)*(cos(al_1)*sin(al_2) +
      cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 +
1089 th_2)*sin(th0_3 + th_3)*sin(al_1)) + a_5*cos(th0_5 + th_5)*(cos(th0_4 +
      th_4)*(cos(th0_3 + th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)) - sin(th0_4 + th_4)*(sin(th0_3 +
1090 th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_3)*sin(al_1))) + a_6*cos(th0_6 + th_6)*(cos(th0_5 +
      th_5)*(cos(th0_4 + th_4)*(cos(th0_3 + th_3)*(cos(al_1)*sin(al_2) +
      cos(th0_2 +
1091 th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)) - sin(th0_4 + th_4)*(sin(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_3)*sin(al_1))) -
1092 sin(th0_5 + th_5)*(cos(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_3)*sin(al_1)) - sin(al_4)*(sin(th0_3 +
1093 th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*cos(al_4)*(cos(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) -
1094 sin(th0_2 + th_2)*sin(th0_3 + th_3)*sin(al_1))) - a_3*sin(th0_2 +
      th_2)*sin(th0_3 + th_3)*sin(al_1) ;
1095

```

```

1096 J_out(3,4)= d_6*(cos(al_5)*(sin(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
1097 th_3)*cos(al_3)*sin(al_1)) + cos(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1))) - sin(th0_5 +
      th_5)*sin(al_5)*(sin(th0_4 + th_4)*(sin(th0_3 +
1098 th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) - cos(th0_4 +
      th_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
1099 th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1))) + cos(th0_5 + th_5)*sin(al_5)*(sin(th0_4
      + th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
1100 th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1)) + cos(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) +
1101 cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*sin(al_1))) + d_5*(sin(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) +
1102 cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1)) + cos(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      cos(th0_3 + th_3)*sin(th0_2 +
1103 th_2)*sin(al_1))) + a_6*sin(th0_6 + th_6)*(sin(al_5)*(sin(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) -
1104 sin(th0_2 + th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1)) + cos(th0_4 +
      th_4)*sin(al_4)*(sin(th0_3 + th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*sin(al_1))) + sin(th0_5 +
1105 th_5)*cos(al_5)*(sin(th0_4 + th_4)*(sin(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +

```



```

1115 th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) - a_5*cos(th0_5 +
      th_5)*(sin(th0_4 + th_4)*(sin(th0_3 + th_3)*(cos(al_1)*sin(al_2) +
      cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 +
1116 th_3)*sin(th0_2 + th_2)*sin(al_1)) - cos(th0_4 +
      th_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
1117 th_3)*cos(al_3)*sin(al_1))) ;
1118
1119 J_out(3,5)= d_6*(cos(th0_5 + th_5)*sin(al_5)*(cos(th0_4 +
      th_4)*(sin(th0_3 + th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*sin(al_1)) + sin(th0_4 + th_4)*(sin(al_3)*(cos(al_1)*cos(al_2) -
1120 cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1))) + sin(th0_5 +
1121 th_5)*sin(al_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3)) +
1122 cos(th0_4 + th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1)) -
1123 sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 + th_3)*(cos(al_1)*sin(al_2) +
      cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2
      + th_2)*sin(al_1)))) + a_6*cos(th0_6 + th_6)*(cos(th0_5 +
      th_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) -
1124 cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3)) + cos(th0_4 +
      th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) -
1125 cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +

```





```

1135 cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
    th_3)*sin(al_1)*sin(al_3)) + cos(th0_4 +
    th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
    th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
    th_3)*cos(al_3)*(cos(al_1)*sin(al_2) +
1136 cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
    th_3)*cos(al_3)*sin(al_1)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
    th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
    cos(th0_3 + th_3)*sin(th0_2 +
1137 th_2)*sin(al_1)))) ;
1138
1139 J_out(3,6)= a_6*cos(th0_6 +
    th_6)*(sin(al_5)*(cos(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) -
    cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
    th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
    th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
1140 th_3)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
    th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
    cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) - cos(th0_4 +
    th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) -
1141 cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
    th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
    th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
    th_3)*cos(al_3)*sin(al_1))) - sin(th0_5 + th_5)*cos(al_5)*(cos(th0_4
    + th_4)*(sin(th0_3 +
1142 th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
    cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) + sin(th0_4 +
    th_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
    th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
1143 th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
    th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
    th_3)*cos(al_3)*sin(al_1))) + cos(th0_5 +
    th_5)*cos(al_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) -
    cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) -
1144 cos(th0_3 + th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
    th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
    th_3)*sin(al_1)*sin(al_3)) + cos(th0_4 +
    th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
    th_2)*sin(al_1)*sin(al_2)) +
1145 cos(th0_3 + th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
    th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
    th_3)*cos(al_3)*sin(al_1)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +

```

```

th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
1146 th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
th_2)*sin(al_1))) - a_6*sin(th0_6 + th_6)*(sin(th0_5 +
th_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
th_3)*sin(al_3)*(cos(al_1)*sin(al_2)
1147 + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*sin(al_1)*sin(al_3)) + cos(th0_4 +
th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*sin(al_2) +
1148 cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
cos(th0_3 + th_3)*sin(th0_2 +
1149 th_2)*sin(al_1))) + cos(th0_5 + th_5)*(cos(th0_4 + th_4)*(sin(th0_3 +
th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) + sin(th0_4 +
th_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
1150 th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1))) ;

1151
1152 J_out(3,7)= 1 ;
1153
1154 J_out(3,8)= cos(al_1) ;
1155
1156 J_out(3,9)= cos(al_1)*cos(al_2) - cos(th0_2 + th_2)*sin(al_1)*sin(al_2) ;
1157
1158 J_out(3,10)= cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*sin(al_1)*sin(al_3) ;

1159
1160 J_out(3,11)= cos(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*sin(al_1)*sin(al_3)) +

```

```

1161 sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 + th_3)*(cos(al_1)*sin(al_2) +
      cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2
      + th_2)*sin(al_1)) - cos(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
1162 th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1)) ;

1163
1164 J_out(3,12)= cos(al_5)*(cos(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) -
      cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
1165 th_3)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) - cos(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) -
1166 cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1))) + sin(th0_5 + th_5)*sin(al_5)*(cos(th0_4
      + th_4)*(sin(th0_3 +
1167 th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) + sin(th0_4 +
      th_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
1168 th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1))) - cos(th0_5 +
      th_5)*sin(al_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) -
      cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) -
1169 cos(th0_3 + th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3)) + cos(th0_4 +
      th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) +
1170 cos(th0_3 + th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
1171 th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*sin(al_1))) ;

```

```

1172
1173 J_out(3,13)= 0 ;
1174
1175 J_out(3,14)= sin(th0_2 + th_2)*sin(al_1) ;
1176
1177 J_out(3,15)= sin(th0_3 + th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*sin(al_1) ;
1178
1179 J_out(3,16)= cos(th0_4 + th_4)*(sin(th0_3 + th_3)*(cos(al_1)*sin(al_2) +
      cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2
      + th_2)*sin(al_1)) + sin(th0_4 +
      th_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) +
1180 cos(th0_3 + th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1)) ;
1181
1182 J_out(3,17)= sin(th0_5 + th_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2)
      - cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
1183 th_3)*sin(al_1)*sin(al_3)) + cos(th0_4 +
      th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
1184 th_3)*cos(al_3)*sin(al_1)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) + cos(th0_5 +
      th_5)*(cos(th0_4 + th_4)*(sin(th0_3 +
1185 th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) + sin(th0_4 +
      th_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
1186 th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1))) ;
1187
1188 J_out(3,18)= sin(th0_6 +
      th_6)*(sin(al_5)*(cos(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) -
      cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +

```

```

th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
1189 th_3)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) - cos(th0_4 +
th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) -
1190 cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1))) - sin(th0_5 + th_5)*cos(al_5)*(cos(th0_4
+ th_4)*(sin(th0_3 +
1191 th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) + sin(th0_4 +
th_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
1192 th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1))) + cos(th0_5 +
th_5)*cos(al_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) -
cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) -
1193 cos(th0_3 + th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*sin(al_1)*sin(al_3)) + cos(th0_4 +
th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)) +
1194 cos(th0_3 + th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
1195 th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
th_2)*sin(al_1))) + cos(th0_6 + th_6)*(sin(th0_5 +
th_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
th_3)*sin(al_3)*(cos(al_1)*sin(al_2) +
1196 cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*sin(al_1)*sin(al_3)) + cos(th0_4 +
th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*sin(al_2) +
1197 cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +

```

```

cos(th0_3 + th_3)*sin(th0_2 +
1198 th_2)*sin(al_1))) + cos(th0_5 + th_5)*(cos(th0_4 + th_4)*(sin(th0_3 +
th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) + sin(th0_4 +
th_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
1199 th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1)))) ;

1200
1201 J_out(3,19)= d_4*(cos(th0_3 + th_3)*sin(al_3)*(sin(al_1)*sin(al_2) -
cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) -
cos(al_3)*(cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_1)*sin(al_3)) -
1202 d_6*(sin(th0_5 + th_5)*sin(al_5)*(cos(th0_4 + th_4)*(sin(th0_3 +
th_3)*(sin(al_1)*sin(al_2) - cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) -
cos(th0_3 + th_3)*sin(th0_2 + th_2)*cos(al_1)) + sin(th0_4 +
th_4)*(sin(al_3)*(cos(al_2)*sin(al_1) + cos(th0_2 +
1203 th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_1)*cos(al_3))) - cos(al_5)*(cos(al_4)*(cos(th0_3 +
th_3)*sin(al_3)*(sin(al_1)*sin(al_2) -
1204 cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(al_3)*(cos(al_2)*sin(al_1) +
cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_2 + th_2)*sin(th0_3
+ th_3)*cos(al_1)*sin(al_3)) + cos(th0_4 +
th_4)*sin(al_4)*(sin(al_3)*(cos(al_2)*sin(al_1) + cos(th0_2 +
1205 th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_1)*cos(al_3)) - sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
th_3)*(sin(al_1)*sin(al_2) -
1206 cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*sin(th0_2 +
th_2)*cos(al_1))) + cos(th0_5 + th_5)*sin(al_5)*(sin(al_4)*(cos(th0_3
+ th_3)*sin(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) - cos(al_3)*(cos(al_2)*sin(al_1) +
1207 cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_1)*sin(al_3)) - cos(th0_4 +
th_4)*cos(al_4)*(sin(al_3)*(cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(al_1)*sin(al_2) -

```

```

1208 cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_1)*cos(al_3)) + sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
      th_3)*(sin(al_1)*sin(al_2) - cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) -
      cos(th0_3 + th_3)*sin(th0_2 +
1209 th_2)*cos(al_1))) - d_2*sin(al_1) + d_5*(cos(al_4)*(cos(th0_3 +
      th_3)*sin(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) - cos(al_3)*(cos(al_2)*sin(al_1) +
      cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +
1210 th_3)*cos(al_1)*sin(al_3)) + cos(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(al_2)*sin(al_1) + cos(th0_2 +
      th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +
1211 th_3)*cos(al_1)*cos(al_3)) - sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
      th_3)*(sin(al_1)*sin(al_2) - cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) -
      cos(th0_3 + th_3)*sin(th0_2 + th_2)*cos(al_1))) -
      d_3*(cos(al_2)*sin(al_1) + cos(th0_2 + th_2)*cos(al_1)*sin(al_2))
1212 - a_6*cos(th0_6 + th_6)*(cos(th0_5 + th_5)*(cos(th0_4 + th_4)*(sin(th0_3
      + th_3)*(sin(al_1)*sin(al_2) - cos(th0_2 + th_2)*cos(al_1)*cos(al_2))
      - cos(th0_3 + th_3)*sin(th0_2 + th_2)*cos(al_1)) + sin(th0_4 +
      th_4)*(sin(al_3)*(cos(al_2)*sin(al_1) + cos(th0_2
1213 + th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_1)*cos(al_3))) - sin(th0_5 + th_5)*(sin(al_4)*(cos(th0_3
      +
1214 th_3)*sin(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) - cos(al_3)*(cos(al_2)*sin(al_1) +
      cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_2 + th_2)*sin(th0_3
      + th_3)*cos(al_1)*sin(al_3)) - cos(th0_4 +
1215 th_4)*cos(al_4)*(sin(al_3)*(cos(al_2)*sin(al_1) + cos(th0_2 +
      th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_1)*cos(al_3)) + sin(th0_4 +
1216 th_4)*cos(al_4)*(sin(th0_3 + th_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*cos(al_1))) - a_4*sin(th0_4 +
      th_4)*(sin(al_3)*(cos(al_2)*sin(al_1) + cos(th0_2 +
      th_2)*cos(al_1)*sin(al_2)) +
1217 cos(th0_3 + th_3)*cos(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
      th_2)*cos(al_1)*cos(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +

```





```

th_3)*cos(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_1)*cos(al_3))) + cos(th0_5 +
1228 th_5)*cos(al_5)*(sin(al_4)*(cos(th0_3 +
th_3)*sin(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) - cos(al_3)*(cos(al_2)*sin(al_1) +
cos(th0_2 + th_2)*cos(al_1)*sin(al_2)) + sin(th0_2 + th_2)*sin(th0_3
+ th_3)*cos(al_1)*sin(al_3)) -
1229 cos(th0_4 + th_4)*cos(al_4)*(sin(al_3)*(cos(al_2)*sin(al_1) + cos(th0_2 +
th_2)*cos(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(sin(al_1)*sin(al_2) - cos(th0_2 +
th_2)*cos(al_1)*cos(al_2)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_1)*cos(al_3)) +
1230 sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 + th_3)*(sin(al_1)*sin(al_2) -
cos(th0_2 + th_2)*cos(al_1)*cos(al_2)) - cos(th0_3 + th_3)*sin(th0_2
+ th_2)*cos(al_1))) + a_3*cos(th0_3 + th_3)*sin(th0_2 +
th_2)*cos(al_1) ;

1231
1232 J_out(3,20)= d_6*(cos(al_5)*(cos(th0_4 +
th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) - cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2))) -
1233 cos(al_4)*(cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 +
th_3)*sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2))) + sin(th0_3 + th_3)*sin(th0_4 +
th_4)*sin(al_4)*(cos(al_1)*cos(al_2) -
1234 cos(th0_2 + th_2)*sin(al_1)*sin(al_2))) - sin(th0_5 +
th_5)*sin(al_5)*(sin(th0_4 + th_4)*(sin(al_3)*(cos(al_1)*sin(al_2) +
cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) - cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)))
1235 - cos(th0_4 + th_4)*sin(th0_3 + th_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2))) + cos(th0_5 +
th_5)*sin(al_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*sin(al_2) +
cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 +
1236 th_3)*sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2))) + cos(th0_4 +
th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) - cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +

```



```

1246 th_2)*sin(al_1)*sin(al_2))) + cos(th0_4 +
      th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2))) + sin(th0_3 + th_3)*sin(th0_4 +
1247 th_4)*cos(al_4)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2))) + a_3*sin(th0_3 +
      th_3)*(cos(al_1)*cos(al_2) - cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) -
      a_4*sin(th0_4 + th_4)*(sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
1248 th_2)*cos(al_2)*sin(al_1)) - cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2))) + a_6*sin(th0_6 +
      th_6)*(sin(al_5)*(cos(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
1249 th_2)*cos(al_2)*sin(al_1)) - cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2))) -
      cos(al_4)*(cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*cos(al_2)
1250 - cos(th0_2 + th_2)*sin(al_1)*sin(al_2))) + sin(th0_3 + th_3)*sin(th0_4 +
      th_4)*sin(al_4)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2))) + sin(th0_5 + th_5)*cos(al_5)*(sin(th0_4
      + th_4)*(sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
1251 th_2)*cos(al_2)*sin(al_1)) - cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2))) - cos(th0_4 + th_4)*sin(th0_3 +
      th_3)*(cos(al_1)*cos(al_2) - cos(th0_2 + th_2)*sin(al_1)*sin(al_2)))
      - cos(th0_5 +
1252 th_5)*cos(al_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2))) + cos(th0_4 +
      th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*sin(al_2)
1253 + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) - cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2))) + sin(th0_3 + th_3)*sin(th0_4 +
      th_4)*cos(al_4)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2))) +
1254 a_4*cos(th0_4 + th_4)*sin(th0_3 + th_3)*(cos(al_1)*cos(al_2) - cos(th0_2
      + th_2)*sin(al_1)*sin(al_2)) ;
1255

```

```

1256 J_out(3,21)= d_6*(cos(th0_5 +
      th_5)*sin(al_5)*(sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) -
      cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
1257 th_3)*cos(al_3)*sin(al_1)) - cos(th0_4 +
      th_4)*cos(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
1258 th_3)*sin(al_1)*sin(al_3))) -
      cos(al_5)*(cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
1259 th_3)*cos(al_3)*sin(al_1)) + cos(th0_4 +
      th_4)*sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
1260 th_3)*sin(al_1)*sin(al_3))) + sin(th0_4 + th_4)*sin(th0_5 +
      th_5)*sin(al_5)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 +
1261 th_2)*sin(th0_3 + th_3)*sin(al_1)*sin(al_3))) -
      d_4*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
1262 th_3)*cos(al_3)*sin(al_1)) -
      d_5*(cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
1263 th_3)*cos(al_3)*sin(al_1)) + cos(th0_4 +
      th_4)*sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
1264 th_3)*sin(al_1)*sin(al_3))) + a_4*sin(th0_4 +
      th_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +

```



```

1273 th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1)) - cos(th0_4 +
      th_4)*cos(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
1274 th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3))) - cos(th0_5 + th_5)*sin(th0_4 +
      th_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) -
1275 cos(th0_3 + th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3))) + a_5*cos(th0_5 + th_5)*sin(th0_4 +
      th_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
1276 th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3)) ;

1277
1278 J_out(3,22)= a_5*sin(th0_5 +
      th_5)*(cos(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
1279 th_3)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) - cos(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) -
1280 cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1))) -
      d_5*(sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
1281 th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3)) + cos(th0_4 +
      th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
1282 th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +

```

```

th_3)*(cos(al_1)*sin(al_2) +
1283 cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
th_2)*sin(al_1)) -
d_6*(cos(al_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2
+ th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
1284 th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*sin(al_1)*sin(al_3)) + cos(th0_4 +
th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
1285 th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1))) +
1286 cos(th0_5 + th_5)*sin(al_5)*(cos(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) -
cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
1287 th_3)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) - cos(th0_4 +
th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) -
1288 cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1))) - a_6*sin(th0_6 +
1289 th_6)*(sin(al_5)*(sin(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
th_3)*sin(al_1)*sin(al_3)) +
1290 cos(th0_4 + th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1)) -
1291 sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 + th_3)*(cos(al_1)*sin(al_2) +
cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2
+ th_2)*sin(al_1))) - cos(th0_5 +
th_5)*cos(al_5)*(cos(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) -
cos(th0_2 +

```



```

1292 th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) +
1293 cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) + cos(th0_3 + th_3)*sin(th0_2 +
      th_2)*sin(al_1)) - cos(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
1294 th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1))) + a_6*cos(th0_6 + th_6)*sin(th0_5 +
      th_5)*(cos(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
1295 th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
1296 cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) - cos(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +
      th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 +
1297 th_2)*sin(th0_3 + th_3)*cos(al_3)*sin(al_1))) ;
1298
1299 J_out(3,23)= a_6*sin(th0_6 +
      th_6)*(cos(al_5)*(cos(al_4)*(cos(al_3)*(cos(al_1)*cos(al_2) -
      cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) - cos(th0_3 +
      th_3)*sin(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) + sin(th0_2 + th_2)*sin(th0_3 +
1300 th_3)*sin(al_1)*sin(al_3)) + sin(th0_4 + th_4)*sin(al_4)*(sin(th0_3 +
      th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) - cos(th0_4 +
      th_4)*sin(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) -
1301 cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
      th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
      th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
      th_3)*cos(al_3)*sin(al_1))) + sin(th0_5 + th_5)*sin(al_5)*(cos(th0_4
      + th_4)*(sin(th0_3 +
1302 th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
      cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)) + sin(th0_4 +
      th_4)*(sin(al_3)*(cos(al_1)*cos(al_2) - cos(th0_2 +

```



```

th_4)*cos(al_4)*(sin(al_3)*(cos(al_1)*cos(al_2) -
1312 cos(th0_2 + th_2)*sin(al_1)*sin(al_2)) + cos(th0_3 +
th_3)*cos(al_3)*(cos(al_1)*sin(al_2) + cos(th0_2 +
th_2)*cos(al_2)*sin(al_1)) - sin(th0_2 + th_2)*sin(th0_3 +
th_3)*cos(al_3)*sin(al_1)) - sin(th0_4 + th_4)*cos(al_4)*(sin(th0_3 +
1313 th_3)*(cos(al_1)*sin(al_2) + cos(th0_2 + th_2)*cos(al_2)*sin(al_1)) +
cos(th0_3 + th_3)*sin(th0_2 + th_2)*sin(al_1)))) ;
1314
1315 J_out(3,24)= 0 ;
1316
1317 return

```