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EVALUATING THE PERFORMANCE OF CONSTRAINT FORMULATIONS FOR MULTIBODY DYNAMICS SIMULATION

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ABSTRACT

Contemporary software systems used in the dynamic simulation of rigid bodies suffer from problems in accuracy, performance, and robustness. Significant allowances for parameter tuning, coupled with the careful implementation of a broad phase collision detection scheme is required to make dynamic simulation useful for practical applications. A geometrically accurate constraint formulation, the Polyhedral Exact Geometry method, is presented. The Polyhedral Exact Geometry formulation is similar to the well-known Stewart-Trinkle formulation, but extended to produce unilateral constraints that are geometrically correct in cases where polyhedral bodies have a locally non-convex free space. The PEG method is less dependent on broad-phase collision detection or system tuning than similar methods, demonstrated by several examples.

Uncomplicated benchmark examples are presented to analyze and compare the new Polyhedral Exact Geometry formulation with the well-known Stewart-Trinkle and Anitescu-Potra methods. The behavior and performance for the methods are discussed. This includes specific cases where contemporary methods fail to match theorized and observed system states in simulation, and how they are ameliorated by PEG.

INTRODUCTION

The simulation of the dynamics of rigid bodies has many applications in a variety of scientific and engineering fields. The demand for fast and accurate simulation is high, as dynamic problems grow larger and larger under limited computation power. The performance of an improvement to the popular Stewart-Trinkle [1] multibody dynamic system formulation is evaluated in this work. This recent multibody dynamics constraint formulation, Polyhedral Exact Geometry [2, 3] (PEG) is compared to widely accepted constraint generation methods. The purpose of this new formulation is to increase simulation performance and accuracy. The accuracy of Polyhedral Exact Geometry is evaluated compared to the Stewart-Trinkle and Anitescu-Potra methods, specifically the position error present in a small planar particle simulation.

Multibody dynamic simulation is used in a broad range of engineering, research, and entertainment fields. Accurate simulation of the motion of machinery is crucial in mechanical design [4–6]. It is particularly useful in designing complex machinery that is expensive to prototype, such as internal combustion engines [7]. Accurate simulation is required for robotics applications, particularly where contact is expected, such as in grasp planning [8]. And virtual reality is more useful and convincing when objects undergo more realistic motion in simulation [9]. Entertainment products such as motion pictures and video games benefit from accurate simulation [10], where realism is enhanced.

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There are many challenges involved in providing a reasonable approximation of physical reality through simulation of rigid bodies [11, 12]. This is particularly a difficult issue when considering contact and friction. This consideration makes a system mathematically nonsmooth, and difficult to simulate accurately. A simulation must use constraints that closely represent the geometry of objects interacting with each other. First, accuracy is lost when representing real world objects as topologically connected vertices, faces, and edges in computer simulation [13–15]. In addition, constraints do not always exactly coincide with this approximated geometric representation, causing further error. These problems can be mitigated through increased accuracy by taking small time steps, or by more accurately mapping the constraints to the geometry. The formulations presented in this paper are of the latter solution.

Time-stepping simulations calculate estimates of system state at discrete times. The simulation starts at an initial state, and then calculates the future system state one time step ahead. The systems states at the next time step are dependent on the solution of the dynamics and constraints using the states of the previous time step. And the simulation proceeds until criteria are met, e.g., reaching a predetermined final time.

Within a time step, two phases are common. First, the unilateral and bilateral constraints, and the time-stepping subproblem are formulated. The constraints are determined by the geometry and kinematics of the simulated system. Second, the dynamics of the system are formulated from the equations of motion, and calculated to produce the derivative states. These derivative states are used to calculate all system states for the following time step.

Popular physics engines such as ODE [16], and Bullet [10] use variations of the Stewart-Trinkle [1, 17] and Anitescu-Potra [18, 19] formulations. Inaccuracies resulting from simplifications of Coulomb friction and geometric modeling of constraints result in undesirable behaviors in simulation that make the use of these popular engines for scientific work challenging [2]. The research presented herein continues the work in [3] and [20].

The mean error is used as a metric in evaluating the performance three formulations. The performance of the Polyhedral Exact Geometry formulation is shown to be more accurate than Stewart-Trinkle and Anitescu-Potra . The higher accuracy allows larger step sizes to be utilized while producing similarly accurate results as Stewart-Trinkle and Anitescu-Potra at smaller step sizes.

METHODS

The formulation of the Polyhedral Exact Geometry method is similar to the Stewart-Trinkle [1,17] formulation, but extended to produce unilateral constraints that are geometrically correct in cases where polyhedral bodies have a locally non-convex free space. Constraints for faces in a polygon model are defined as halfspaces in the Stewart-Trinkle and Anitescu-Potra methods. These halfspaces extend beyond the limits of the faces, producing constraints which are not geometrically accurate.

The Polyhedral Exact Geometry method uses a heuristic to choose which constraints are active based on local geometry. This increases the accuracy of the simulation, and eliminates the need for specialized methods of broadphase collision detection required to avoid undesired behaviors in simulation caused by active constraints that should be ignored.

Equations of Motion

Rigid bodies in simulation move in accordance with the equations of motion. In the systems considered in this paper, the Newton-Euler equations are used in the dynamics formulation,

$$\mathbf{M}(\mathbf{q},t)\dot{\nu} = \lambda_{\nu p}(\mathbf{q},\dot{\mathbf{q}},t) + \lambda_{app}, \tag{1}$$

where $\lambda_{\nu p}$ is the sum of velocity dependent forces, and λ_{app} are the applied forces. The system is subject to the complementarity constraint,

$$0 \leq \lambda_n^{l+1} \perp \mathbf{G}_n^T \nu^{l+1} + \frac{\Psi_n^l}{h} + \frac{\partial \Psi_n^l}{\partial t} \geq 0, \tag{2}$$

where λ_n are forces normal to contact surfaces, Ψ_n are gap functions, or distances between active bodies and contact surfaces, h is the length of the time step, ν are the velocities of the active bodies, and $\mathbf{G}_n$ is a normal contact wrench.

Broad phase collision detection

A broad phase collision detection scheme is utilized to avoid the need to include all possible contact constraints at every time step. In the specific implementation discussed herein, if any edge in a body is closer than a predetermined value ϵ to the particle, then all edge constraints in that body are added to the formulation for the current time step. ϵ can be calculated as a function of particle velocity, or tuned for specific systems. The inclusion of the tunable parameter ϵ can cause undesirable behavior when used in conjunction with the Stewart-Trinkle method. Objects might pick up additional kinetic energy, or have an incorrectly modified trajectory from interacting with constraints extended beyond the actual object boundaries.

Stewart-Trinkle Formulation of the Mixed Linear Complementarity Problem

Equations (1) and (2) are formulated as a mixed linear complementarity problem [21, 22], using the Stewart-Trinkle formulation,

$$\left|\begin{matrix}0\\ \rho_n^{l+1}\end{matrix}\right| = \left|\begin{matrix}M & -G_n\\ G_n^T & 0\end{matrix}\right| \left|\begin{matrix}\nu^{l+1}\\ p_n^{l+1}\end{matrix}\right| + \left|\begin{matrix}-M\nu^l - p_{ext}^l\\ \psi_n^l/h\end{matrix}\right|, \tag{3}$$

where M is the block diagonal generalized inertia matrix of all active bodies in the system, ν is the generalized velocities, p_n and p_{ext} are normal and external impulsive forces, and h is the time step. The normal contact wrench is defined as

$$G_{n_{ij}} = \begin{bmatrix}\hat{n}_{ij}\\ r_{ij} \times \hat{n}_{ij}\end{bmatrix}, \tag{4}$$

which is indexed over k collisions by the i^{th} body and j^{th} collision. Friction is omitted from the formulation for clarity. The term r is the position vector from the center of mass of the body to the point on the body in collision with another object. $\hat{n}$ is a unit vector in the direction normal from the surface in collision.

The problem size for the Stewart-Trinkle method is $2 \cdot b + c$, where b is the number of active bodies, and c is the number of contacts [1].

When considering friction, the formulation in (3) becomes,

$$\left|\begin{matrix}0\\ \rho_n^{l+1}\\ \rho_f^{l+1}\\ \sigma^{l+1}\end{matrix}\right| = \left|\begin{matrix}M & -G_n & -G_f & 0\\ G_n^T & 0 & 0 & 0\\ G_f^T & 0 & 0 & E\\ 0 & U & -E^T & 0\end{matrix}\right| \left|\begin{matrix}\nu^{l+1}\\ p_n^{l+1}\\ p_f^{l+1}\\ s^{l+1}\end{matrix}\right| + \left|\begin{matrix}-M\nu^l - p_{ext}^l\\ \Psi_n^l/h\\ 0\\ 0\end{matrix}\right| \tag{5}$$

where each of the submatrices are defined in terms of the bodies found to be in collision, indexed over k collisions by the i^{th} body and j^{th} collision, i.e.

$$M = \mathrm{blockdiag}(M_1, ..., M_{n_c})$$

where $M_i = \begin{bmatrix}m_i I_{(3\times3)} & \mathbf{0}\\ \mathbf{0} & J_{i(3\times3)}\end{bmatrix}$ and n_c is the number of contacts. The element

$$G_{f_{ij}} = \begin{bmatrix}\hat{d}_{ij1} & ... & \hat{d}_{ijn_d}\\ (r_{ij} \times \hat{d}_{ij1})_Z & ... & (r_{ij} \times \hat{d}_{ijn_d})_Z\end{bmatrix}$$

where n_d is the number of friction directions, and

$$U = \mathrm{diag}(\mu_1, \ldots, \mu_k),$$

with

$$E = \mathrm{blockdiag}\left(\left|\begin{matrix}1\\ 1\\ \vdots\end{matrix}\right|_1, \ldots, \left|\begin{matrix}1\\ 1\\ \vdots\end{matrix}\right|_k\right)$$

where each column vector of ones has length equal to the number of friction directions in the friction cone.

For each body P_i involved in a collision, ν_i^{l+1} is extracted from a solution of the MCP and used to update the states of the body,

$$\nu_i \leftarrow \nu_i^{l+1}, \tag{6}$$

where the configuration parameters u are defined by

$$u_i \leftarrow u_i + h\nu_i^{l+1}, \tag{7}$$

and for bodies not in contact,

$$\nu_i \leftarrow \nu_i + h\frac{F_{ext_i}}{m_i}, \tag{8}$$

and

$$u_i \leftarrow u_i + h\nu_i. \tag{9}$$

General Polyhedral Exact Geometry Formulation

A general system with m facets in contact is realized,

$$
\begin{aligned}
0 \le c_2 - \psi_2 + \psi_1 \perp c_2 \ge 0 \quad (10)\\
0 \le c_3 - \psi_3 + c_2 + \psi_1 \perp c_3 \ge 0\\
\vdots\\
0 \le c_m - \psi_m + c_m + c_{m-1} + \ldots + c_2 + \psi_1 \perp c_m \ge 0\\
\\
0 \le d_1 + \psi_1 \perp d_1 \ge 0\\
0 \le d_2 + \psi_2 \perp d_2 \ge 0\\
\vdots\\
0 \le d_m + \psi_m \perp d_m \ge 0\\
\\
0 \le d_1 + (c_2 + c_3 + \ldots + c_{m-1} + c_m) + \psi_1 \perp \lambda_1 \ge 0\\
0 \le d_2 + (c_2 + c_3 + \ldots + c_{m-1} + c_m) + \psi_2 \perp \lambda_2 \ge 0\\
\vdots\\
0 \le d_m + (c_2 + c_3 + \ldots + c_{m-1} + c_m) + \psi_m \perp \lambda_m \ge 0\\
\\
0 \le (c_2 + c_3 + \ldots + c_{m-1} + c_m) + \psi_1,
\end{aligned}
$$

where

$$c_i = max(0, \psi_1 - \psi_i), i = 2, \ldots, m, \quad (11)$$

and d are slack variables. Note that $\psi_i := [\psi_n]_i$. See [3] for further details of this derivation.

Polyhedral Exact Geometry Formulation of the Mixed Linear Complementarity System

The original Stewart-Trinkle forumation in (3) is altered to introduce a heuristic method based on the Polyhedral Exact Geometry formulation discussed in Section . The main formulation becomes

$$
\begin{vmatrix} 0 \\ \rho_n^{l+1} \\ \rho_a^{l+1} \end{vmatrix} = \begin{vmatrix} M & -G_n & 0 \\ G_n^T & 0 & E_1 \\ G_a^T & 0 & E_2 \end{vmatrix} \begin{vmatrix} \nu^{l+1} \\ p_n^{l+1} \\ c_a^{l+1} \end{vmatrix} + \begin{vmatrix} -M\nu^l - p_{ext}^l \\ \Psi_n^l / h \\ \Delta\Psi_a / h \end{vmatrix}, \quad (12)
$$

where M is the generalized inertia matrix, and G_n and G_a are the normal and auxiliary contact wrenches, respectively. Multiple adjacent contacts are grouped, and the components of G_n from (4) are split into G_n and G_a in (12). G_n contains the contact wrenches of the contacts with the minimum value of ψ_n for each group of contacts. The remaining contact wrenches in each manifold are put into G_a. The auxiliary gap functions are defined as

$$
\Psi_a = \begin{bmatrix} \Psi_{a_1} \\ \vdots \\ \Psi_{a_{n_s}} \end{bmatrix} \text{ where } \Psi_{a_j} = \begin{bmatrix} \Psi_1 - \Psi_2 \\ \vdots \\ \Psi_1 - \Psi_{n_s} \end{bmatrix}. \quad (13)
$$

This employs a heuristic method, where only the closest constraint with a positive distance ψ_n is chosen as the active constraint. Further constraints are required to guarantee that only one constraint in each group of contacts is active at each instant. One normal contact wrench is put in $\mathbf{G}_n$ for each manifold, while all the other contact normals are put in the auxiliary wrench. The selection matrices apply the additional constraints to enforce that the correct constraint is active.

All elements of the component of the selection matrix E_{1_j} are equal to one, for each contact. The selection matrix E_2 is such that all E_{2_j} are equal to a lower triangular matrix where all nonzero values are equal to one.

The resulting size of the MCP system for the Polyhedral Exact Geometry method is $2 \cdot b + c$, where b is the number of active bodies, c is the total number of contacts.

RESULTS

The RPI MATLAB Simulation Testbed [23] is used to evaluate the performance of three formulation methods. The testbed is designed to evaluate multiple formulations and solvers within a single standard framework. A particle is simulated to move in planar space in the presence of obstacles comprised of line segments. A system of repeating edges arranged in a sawtooth pattern is used for the analysis presented herein. The particle is given an initial position and velocity, then released to move with the force of gravity as the only external force.

TABLE 1. Initial conditions for all simulations

Mass	5	kg
Position X	0	m
Position Y	0.75	m
Velocity X	6	m / s
Velocity Y	1.8	m / s
Static Coefficient of Friction	0.5	
Dynamic Coefficient of Friction	0.2	

The particle simulation with initial conditions as in Table 1 is run, and the resulting simulated trajectories are shown in Figure 1.

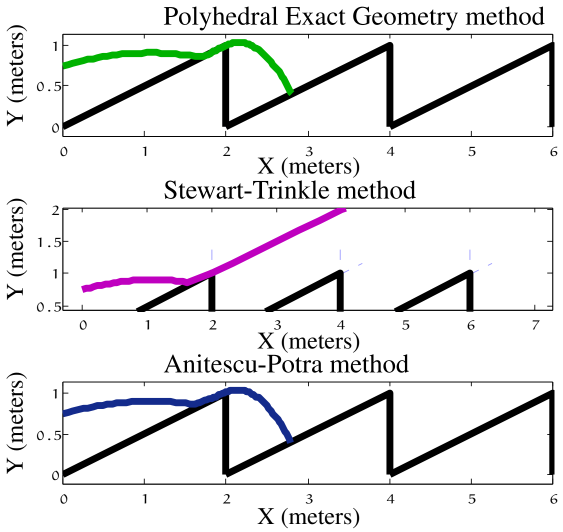

FIGURE 1. Trajectory comparison.

A detailed plot of the trajectories from Figure 1 is given in Figure 2. This highlights the differences present in the trajectories resulting from each method near where the particle interacts with the unilateral constraint from the sloped segment of the first sawtooth. The Polyhedral Exact Geometry method exhibits the correct behavior as verified by an analytical solution of the equations of motion. The particle in this case slides up the first edge, and then leaves for the second phase of ballistic motion.

The Stewart-Trinkle method shows the most error in the trajectory. The particle initially interacts with the constraint of the sloped edge. Constraints in this simulation are active if they are within a configured distance ϵ. As the particle slides up the slope, the constraint from the vertical edge becomes active suddenly. The constraint is violated, meaning that the particle is in penetration. A large impulse force results, which pushes the particle out at a high velocity. The Anitescu-Potra method behaves similar to the Stewart-Trinkle method, except that the impulsive force is not present when the constraints are active. This causes the trajectory to divert at distance ϵ near the sloped constraint.

The sawtooth particle simulation is run multiple times with random initial states. The simulation is run using each of the three methods, starting with a small step size of 1×10^{-5}. This step size is doubled for subsequent simulations over 15 iterations until reaching a value of 0.3277.

The mean error of 62 total simulation runs with random initial states is plotted in Figure 3. The Polyhedral Exact Geometry method has two orders of magnitude less error compared to the Stewart-Trinkle method at each step size. This shows that the Stewart-Trinkle and Anitescu-Potra require a much smaller step size to achieve the same accuracy as the Polyhedral Exact Geometry method.

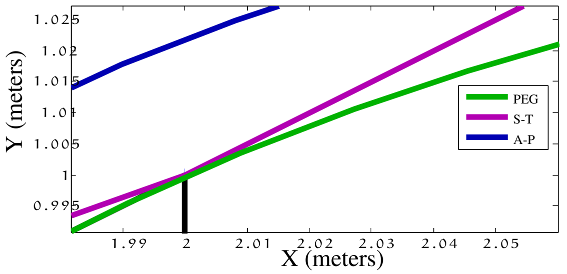

FIGURE 2. Trajectories in proximity to the beginning of the second ballistic phase.

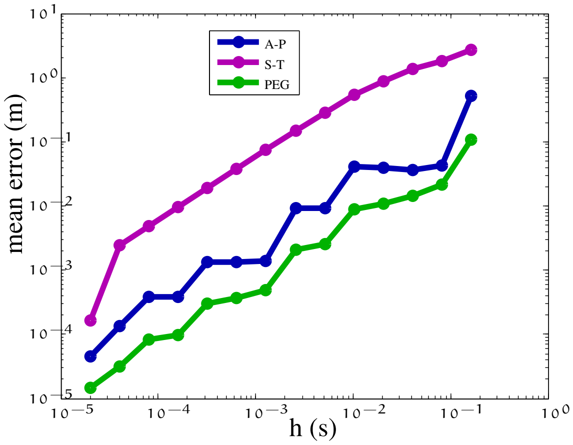

FIGURE 3. Mean error for each method versus step size.

For example, the Polyhedral Exact Geometry method can be used with a step size of 1×10^{-2}, yet still achieve the same accuracy as the Stewart-Trinkle method with a step size of approximately 1×10^{-4}. A significant speedup is realized here, as the problem sizes are similar for each formulation. This performance increase scales up to the general three dimensional problem consisting of many polyhedral bodies.

CONCLUSION

The performance of three multibody dynamic system formulations was compared, showing that performance gains are realized on small benchmark problems by using a geometrically exact method. The Polyhedral Exact Geometry method allows the use of larger step sizes while achieving similar accuracy to Stewart-Trinkle and Anitescu-Potra formulations run under significantly smaller step sizes.

Future work includes performing continuing evaluation of Polyhedral Exact Geometry with larger problem sizes, and comparisons against other common formulation methods.

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